

PRACTICAL EXERCISES IN LIGHT

BEING A LABORATORY COURSE FOR
SCHOOLS OF SCIENCE AND COLLEGES

BY

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MACMILLAN AND CO., LIMITED
ST. MARTIN'S STREET, LONDON

1926

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First Edition, 1902

Reprinted, with corrections, 1906, 1910, 1914, 1920, 1926

PRINTED IN GREAT BRITAIN

PREFACE

THE practical exercises in Light contained in this little book have grown out of laboratory notes prepared from time to time for my own students.

The subjects dealt with in the following pages will be found sufficient to meet the requirements of candidates for the Advanced Stage examination in Light of the Board of Education, South Kensington, and with a little supplementing will prove enough for students taking the Pass B.Sc. examinations of the University of London.

With the single exception of the spectrometer, the apparatus is of the simplest description, and, with the instructions supplied, students and teachers should have no difficulty in preparing it for themselves. Since a large number of sets of the apparatus, necessary for the experiments described, can be made at a small cost, the study of Light should become more popular than it is in Schools of Science and other institutions where Physics is studied practically.

The illustrations are numerous, and have all been drawn specially for this book. At the end of nearly every chapter, additional exercises have been provided, and very few instructions are given for their performance; this plan will enable students working alone to see if they have really understood the typical experiments they have performed with the assistance of the text, and will provide the teacher with a suitable means of keeping the better students usefully occupied while the

slower members of the class are completing the fundamental exercises.

I am glad to thank Mr. F. Castle for permission to use the table of sines, and Mr. H. E. Hadley for allowing me to include the table of tangents at the end of the book ; my thanks are also due to Prof. R. A. Gregory and Mr. A. T. Simmons for much valuable advice and criticism while the book was passing through the press, and to Mr. H. Waite for his assistance in correcting the proofs.

Any suggestions that would help to make the book more useful, or any hints for new experiments in Light, would be gladly welcomed from teachers using the exercises in their laboratories.

REGINALD S. CLAY.

TUFNELL PARK, N.

November, 1902.

PREFACE TO THE 1914 EDITION

As a result of the incorporation of the greater part of this book in my larger "Treatise on Practical Light," some errors have been discovered in the former editions. These have been corrected, and I hope the text is now accurate.

R. S. C.

ESKDALE, FORTIS GREEN, N. 2.

December, 1913.

PREFACE TO THE SIXTH EDITION

SOME further experiments on Photometry have been added to illustrate this important subject.

R. S. C.

ESKDALE, FORTIS GREEN, N. 2.

August, 1926.

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PRACTICAL EXERCISES IN LIGHT

CHAPTER I

PIN EXPERIMENTS

1. Laws of Reflection of Light.

Apparatus.—Drawing-board and paper, plane mirror, that can be supported in a vertical plane, about 3 inches broad by 1 inch high, which must be of good glass—preferably “patent plate” (the great thickness of ordinary plate-glass is a disadvantage).

i. **Equality of Angles of Incidence and Reflection.**—Draw a straight line, AB, on the paper. Stand the mirror on the paper so that its silvered surface is vertically over this line. Stick a pin, Q, in the paper close to the front surface of the mirror, and another, P, as far as the paper will allow distant from it in an oblique direction. Look into the mirror along the line PQ joining these two pins so that the one, Q, nearest the mirror is hidden by the pin P.

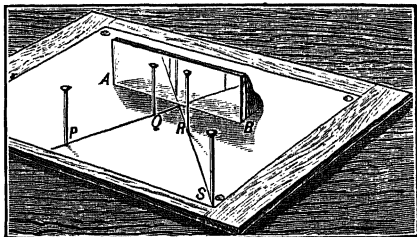


FIG. 1.—Reflection in a Plane Mirror.

Without moving the head insert two more pins, R and S, one close to the surface of the mirror and one a long way off, in such positions that their images appear in the mirror along the continuation of the line PQ.

If the head be moved so as to look along the line SR, the images of Q and P will be found to appear in continuation of the line SR.

Remove the mirror, join PQ and SR, produce them to meet the line which coincided with the silvered surface of the mirror.

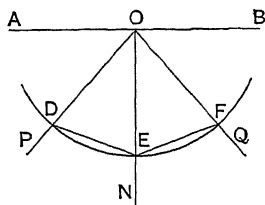


FIG. 2.—Law of Reflection.

They should intersect approximately at O on this line. With a set square draw a line, ON, through O perpendicular to AB. Then the angles PON, NOS should be equal.

With O as centre, and radius of about 12 cms., describe an arc of a circle cutting PQ, ON, RS, in D, E, F.

Measure the distance DE, EF with a millimetre scale. Copy this figure on a reduced scale in your notebook, and enter these distances. The equality of these distances will measure the accuracy with which the experiment has been performed.

ii. **Equality of Distance of Image and Object from the Reflecting Surface.**—(a) *Sighting Method.*—Draw a line, AB, upon the paper, and place the mirror so that this line may coincide with its silvered surface. About 6 cms. in front of the mirror place a pin, Q. To find the position of the image, q , of this pin, insert pins, P_1 , P_2 , in such positions that they seem in a line with this image, so that looking along the line P_1P_2 , the pin P_1 appears to hide both P_2 and q , the image of Q. In the same way, insert a pair of pins, P_3 , P_4 , in some other direction. Remove the mirror, join P_1P_2 and P_3P_4 , and produce these lines till they meet at q . Join qQ . The line qQ should be at right angles to AB and be bisected by it. Actually measure the distances of q and Q from this line and insert them in your notebook.

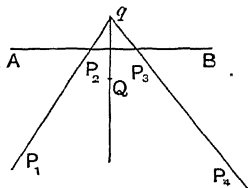


FIG. 3.—Position of Image.

(b) *Parallax Method*.—Place the mirror and pin Q as before. Looking in the mirror nearly normally, with the eye at such a height that only the lower part of the image of Q is visible in the mirror, place a pin, Q', in the paper at the back of the mirror

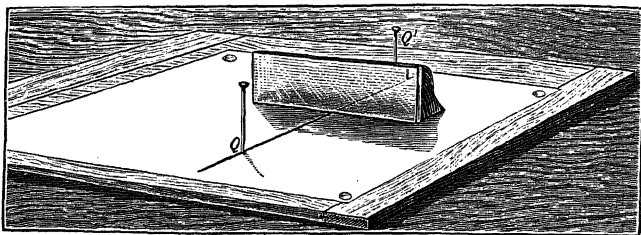


FIG. 4.—Position of Image by Parallax. The image of Q appears to the right of Q', and is therefore behind Q'. Q' is to be moved farther from the mirror.

so that the upper part of this pin may seem to be a continuation of the image q of Q. Q' is evidently then on the line joining the eye and the image q ; it may be in front of or behind this image.

To determine this, move the eye to the right as far as the mirror will allow. If Q' still appears to be the continuation of the image of Q it has been so placed as to coincide with q exactly; but generally it will be found to separate from q . If it appears to have moved to the right of q (*i.e.* to have moved in the same direction as the eye has been moved), it is too far away from the mirror and must be brought a little nearer. By repeated trials a position for Q' will be found such that, wherever the eye be placed, the image of Q and pin Q' will appear to be continuous. If the distances of Q and Q' from the line AB be now measured, they should be found equal. Their actual lengths must be taken and entered in the notebook, in which a reduced drawing should be made.

Note on the Parallax Method.—We are accustomed to judge of the position of ordinary objects by two methods: firstly *the apparent size*, and secondly what we may call *the stereoscopic method*. In the first, knowing the actual size of the object, we are able by accumulated experience, very accurately to estimate the actual distance by the angle the object subtends at the eye; *i.e.* by its apparent size. In the

second method, if we can see the object with both eyes we can estimate its distance by the convergence of the axes of the eyes.

In addition to these two methods, we can to some extent judge the distance by the change in the *accommodation* of the eye, and by observing its *position* relative to that of objects at a known distance.

When, however, an image is formed by a lens or mirror, most of these methods fail us. As the lens or mirror is usually not large enough to allow the object to be seen simultaneously by both eyes, we are unable to judge of its position by the convergence of the axes, and for near objects this is probably the one upon which we mostly rely in our everyday experience. The size of the image being as a rule quite different from that of the object, it is obvious that we shall be unable to judge its distance by its apparent size, since we have no means of estimating it. Indeed, as it is very difficult to dissociate the image from the object, the size of which we really know, the apparent size makes the distance of the image appear very different from its true distance and produces a powerful optical illusion; an object seen through a concave lens, for instance, appears to be at a very much greater distance than it really is, since its apparent size is less than the size of the original; and students frequently find it difficult to realize the fact that the image is really closer to them than the object. This illusion is so powerful that it more than neutralizes the estimate of the distance that should be furnished us by the accommodation of the eye. We are therefore left with only one method by which we can, without further apparatus, judge of the distance of the image—namely, its position relatively to other objects. It is this method which is referred to as *the method of parallax*.

If an observer will stand in front of a window, and between him and the window there is some object such as a retort stand or hanging gas lamp, if he moves sideways, say to the left, he will find that the retort stand appears to move across the window in the opposite direction—namely, to the right. On the other hand, if he looks through the window at some object outside, and moves to the left, he will find that the object moves across the window in the same direction—namely, to the left. So that an object which is further from him than the window (outside it) will appear to move across the window in the same direction as the observer moves, to the left when he moves to the left, to the right when he moves to the right. Exactly the opposite occurs when the object is within the room, *i.e.* nearer to him than the window; it moves across the window to the right when he moves to the left, and *vice versa*.

To apply this, let us suppose that a lens is placed in front of some object—a convex lens, for instance, at a distance of 2 or 3 feet

from a sheet of newspaper, or better still, a poster. Look into the lens from some little distance. An image of the poster will probably be visible in the lens. Sometimes it will be erect and at others inverted,

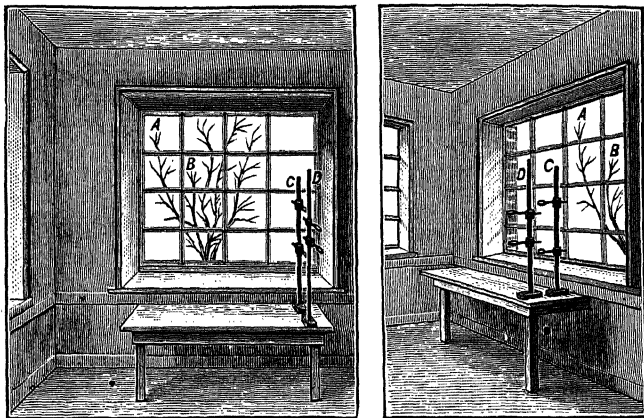


FIG. 5.—*Effect of Parallax.* When the observer has moved to the right, C and D, which are within the room, have moved across the window to the left; A and B have moved to the right. C, which is further from the observer, has moved to the right of D (although both have moved to the left relative to the window).

depending upon the focal length and the distance of the object. Thinking of this lens as a window, it is obvious that if the image is on the far side of the lens, when the observer moves to the left, the image of the poster will move across the lens to the left also. Whilst if the image is on the observer's side of the lens, a movement of the observer to the left should cause it to appear to move across the lens to the right. By observing whether the movement of the image of the poster across the lens is in the same direction as the observer's own movement, or whether it is in the opposite direction, he can therefore ascertain if the image is formed on the far side of the lens or on his side of the lens.

Again, suppose there to be two objects at different distances outside the window, practically in a line with one another. If the observer moves to the left both of these will move across the window to the left, but it will be found that they no longer remain in line with one another—the further one moves to the left across the window by a greater amount than the nearer one, so that the further one moves to the left of the nearer one.

If he tries this with objects within the room, say two retort stands, placed in a line between him and the window, when he moves to the left both retort stands will move across the window to the right, but they will not appear to move by equal amounts—the retort stand which is further from him moving less across the window to the right than the one nearer to him; so that, considering the retort stands alone, the further retort stand is now apparently to the left of the nearer one. Therefore, when we have two objects either without or within the room at different distances from the observer (and whether, therefore, the objects appear to move across the window in the same direction as the observer or the opposite one), in all cases **the further object of the two will appear to move, relatively to the nearer object, in the same direction as the observer.**

This rule should be firmly impressed on the memory.

2. Laws of Refraction.

Apparatus.—A half plate cutting-shape,¹ drawing-board, paper, pins, and millimetre scale.

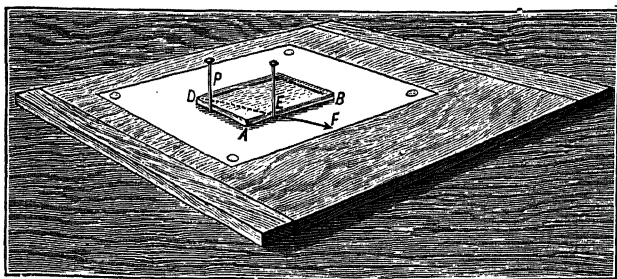


FIG. 6.—Refraction from a Glass Plate into Air.

i. **The Ratio of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction, for any one Medium, is a Constant, and is called the Refractive Index.**—(a) Draw a line on the paper and place the edge, AB, of the plate upon this line. Looking through the surface AB, in a line with a pin, E, observe the corner D, and place a pin, P, in such a position close to the

¹ The glass plate used by a photographer to trim his prints. It must have rectangular edges, one with bevelled edges will be useless.

edge AD, that the corner D, as seen, through the block, may seem to be continued above the block by the pin P. Then the ray DE on emergence traverses along a line EF which is a continuation of the line PE. The refractive index is the ratio $\frac{ED}{EP}$: for by drawing the normal at E it is easy to see that the angles EPA and EDA are equal to the angles of incidence and refraction, NEF and N'ED, respectively, and therefore—

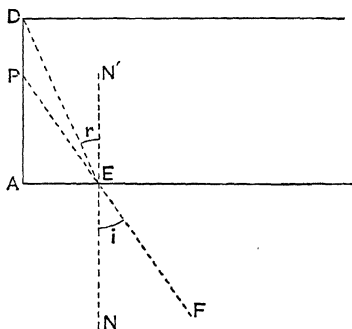


FIG. 7.—Refractive Index of a Glass Plate.

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin EPA}{\sin EDA} = \frac{\frac{AE}{EP}}{\frac{AE}{ED}} = \frac{ED}{EP}.$$

Repeat the experiment with the pin E in a different position, and again find the ratio $\frac{ED}{EP}$; see that the two values of μ are the same, or nearly so.

Copy the figures in your notebook on a reduced scale, and fill in the distances you measure.

(b) Draw a line on the paper and place the edge AB of the block over it. Close to the edges CD and AB insert pins P and Q so that the line PQ may be oblique to the face AB. Holding the head down so that P is seen through the block and closing one eye, place yourself in such a position that the pin Q is in a line with (and therefore covers) the image of P as seen through the glass. (To make sure that beginners are really viewing the pin P through the block, it is helpful to put a piece of wood on the upper surface of the glass, so that the pin P can only be seen through the glass itself. To make sure that the student is looking at P, and not at the image of one of the corners, C or D, the pin may be rocked slightly and the student

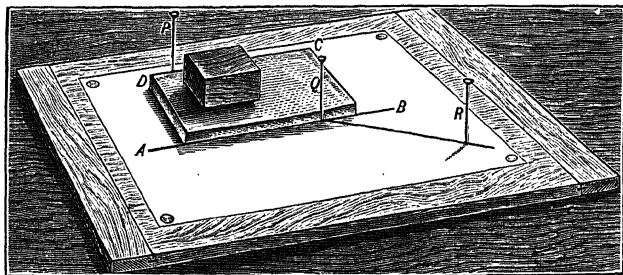


FIG. 8.—Refraction through a Glass Plate.

told to observe that its image, at which he is looking, moves also.) Now insert at a distance of 10 or 12 cms. a third pin, R, so that it may appear in a line with Q and this image of P.

Remove the block, draw the normal MM' . With Q as centre, describe a circle of about 8 cms. radius, cutting QP , QR in K and L. Drop perpendiculars KM' , LM , and measure their lengths.

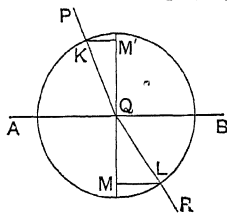


FIG. 9.—Refractive Index.

The refractive index, μ , is given by $\frac{LM}{KM'}$.

Replace the block, and the pin Q, and repeat the experiment for a new position of R. See that the value of μ now obtained agrees with the former one.

Copy the figure about one-third size in your notebook, and insert the actual lengths found for KM' , LM , in each case.

ii. When a Ray of Light

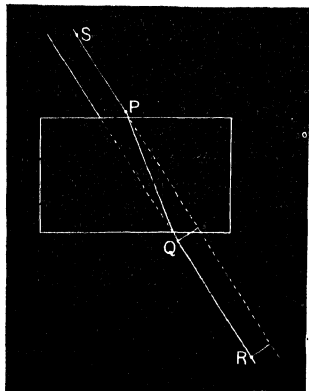


FIG. 10.—Path of a Ray through a Parallel Glass Plate.

passes obliquely through a Block of Glass with Parallel Sides, the Incident and Emergent Rays will be Parallel.—Place the block of glass on paper, and run a fine pencil line along the surfaces AB, CD. Insert the pins P, Q, R, as directed in the last experiment. Then looking in through the other side of the block, insert another pin in S, so that S, P, Q seem in a line. Remove the block, produce the line SP, and drop perpendiculars on it from Q and R.

Measure these perpendiculars : they should be equal.

Make a reduced copy of the figure in your notebook, and insert the actual lengths of the lines.

3. Refraction Experiments with a Prism.

Apparatus.—Glass prism,¹ drawing-board, paper, pins, scale, and protractor.

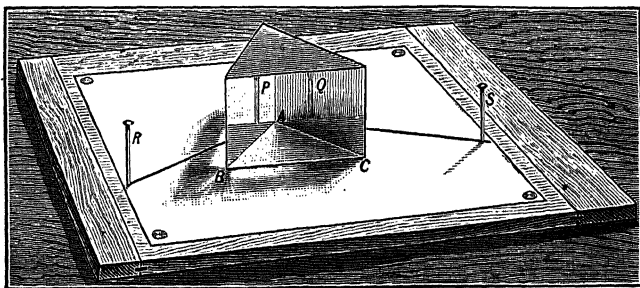


FIG. 11.—Refraction through a Prism.

i. **Symmetrical Refraction.**—Place the prism on the paper, and with a fine pencil draw a line along the edges AB and AC. Remove the prism for a moment, and insert pins at P and Q, so that the triangle APQ may be roughly isosceles. Replace the prism, seeing that its edges exactly coincide once more with the lines AB, AC. Looking in the prism in the direction RP, with one eye closed, place yourself so that P is in line with the image of Q, and insert a pin, R, as far from P as the paper

¹ Large prisms of about 3-inch face can be obtained at a low price for this experiment from Messrs. Pye and Co., Mill Lane, Cambridge.

will allow. Then, looking in the direction SQ , insert another pin to appear in a line with Q and the image of P as seen through the prism. Remove the prism. Join the pins R, P , by a line, and produce this line to F , cutting the line AB at D . In

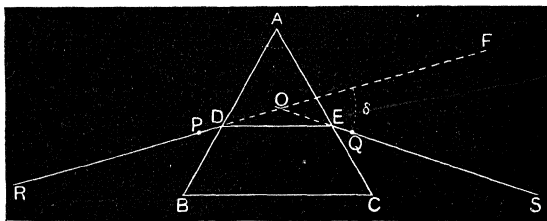


FIG. 12.—Refraction through a Prism.

the same way join S, Q , by a line, and produce it to meet the other one at O , cutting the surface AC at E .

Measure the angle FOS .

If we imagine the ray to be incident along the line RP , it passes through the prism in the direction DE and emerges along ES . If the prism had not been there, this same ray would have proceeded along OF ; thus, the prism has bent the ray from the direction OF to the direction OS , and the angle FOS measures the *deviation* which the prism has produced.

Find this angle with the protractor, and also the angle BAC . Copy the figure into your notebook, and enter the magnitudes of these angles.

Observe that, when looking along SQ to insert the pin S , the image of R , seen in the distance, appears somewhat indistinct and coloured on emergence, blue on one side and red on the other. This is because the blue and red rays are unequally bent by the prism. It is on this account that you are advised to insert S looking along the line SQ , instead of along the line RP ; for it is difficult to say exactly where S should be placed if it is observed through the prism.

ii. **The Path through a Prism of a Ray which has been internally reflected.**—Begin by drawing the lines AB, AC , and BC , and insert the pins P and Q as in the last experiment. Looking in the direction RP , an image of Q will be seen which has been

formed by reflection at the surface BC. That it is the image of Q can be verified by slightly rocking the pin, when the image will be seen to move also. Insert the pin R as far from P as

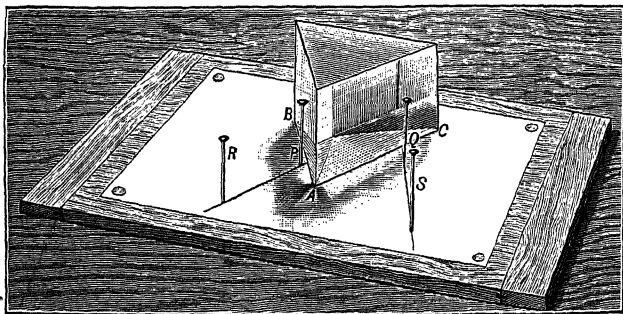


FIG. 13.—Reflection in a Prism.

possible to appear in a line with P and this image. In the same way, the pin S may be inserted by looking along the line SQ. But as it will be found that S is not coloured this time it may be equally well inserted by looking along RP.

[If two students are working together on this experiment, one may look in the direction RP and the other in the direction SQ ; thus they may check each other.]

Remove the block, join RP and produce it to meet AB in D, and SQ to meet AC in E. To obtain the direction of the ray DFE in the block, we must remem-

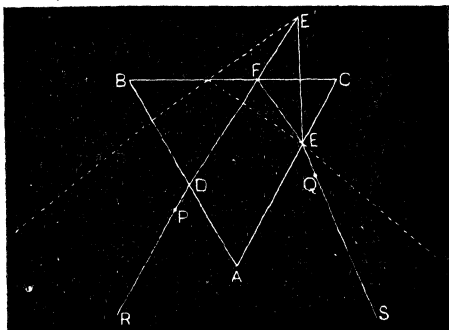


FIG. 14.—Reflection in a Prism.

ber that it has been regularly reflected at F , and therefore that the image of any point such as E will be formed at E' at an equal distance on the other side of BC . Therefore drop a perpendicular EM and produce it, making $ME' = ME$. Join DE' cutting BC at F . Join FE . Then EFD is the path of the ray in the prism.

iii. **Determination of the Angle of a Prism.**—The following experiment, although not giving good results, is useful as

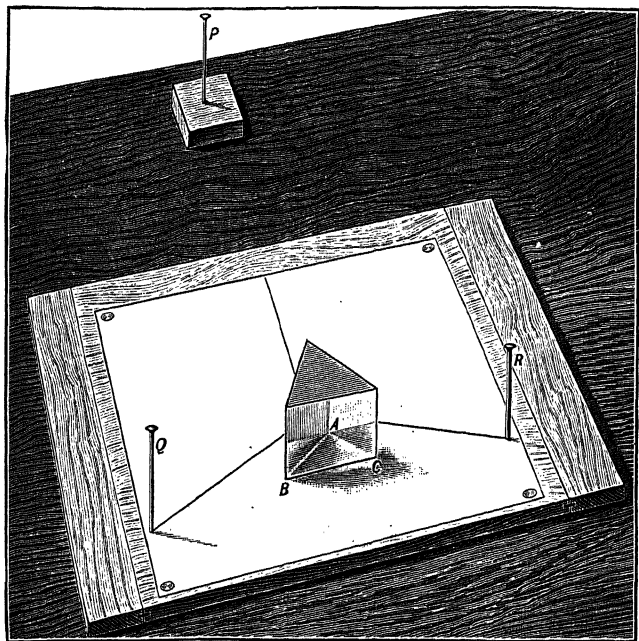


FIG. 15.—Angle of a Prism.

illustrating the method which is used for determining the angle with the spectrometer.

Place the prism on the paper and draw the lines AB and AC . Insert the pin P as far away from A as possible (on a separate

block at a distance) so that the line PA is approximately equally inclined to AB and AC ; and as far away as possible insert a pin, Q, in such a position that looking along the line QA the image of P in the surface BA may be formed as close as possible to the angle A ; that is, so that the pin Q and the image P' may be on the line passing through the corner A. In the same way insert the pin R so that the image of P formed in the surface AC may be as near the corner A as possible.

Remove the prism. Join BA, QA, RA. The angle QAR will be double the angle of the prism, BAC.

To prove this, produce the line PA to L. Then, as the lines AP and AQ are equally inclined to the surface AB—

$$\angle LAB = \angle BAQ,$$

and therefore—

$$\angle LAQ = \text{twice } \angle LAB.$$

In the same way $\angle LAR = \text{twice } \angle LAC$, so that $\angle QAR = \text{twice } \angle BAC$.

Copy the figure in your notebook on a reduced scale, and insert the actual values as found by the protractor of the angles QAR and BAC.

4. Caustic by Refraction at a Plane Surface.

Apparatus.—A half plate cutting - shape¹ with rectangular edges, a drawing-board, pins, and set square.

Lay the plate down flat on a piece of drawing paper and draw a line along one edge of the glass. Place a pin, P, against the opposite long edge near one corner. Looking through the block at this

¹ The glass shapes used by photographers to trim their prints (they sometimes have bevelled edges, but these would be useless for this experiment).

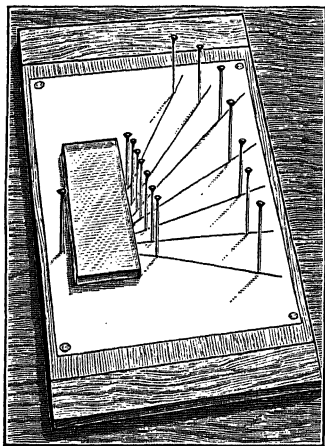


FIG. 16.—Caustic by Refraction at a Plane Surface.

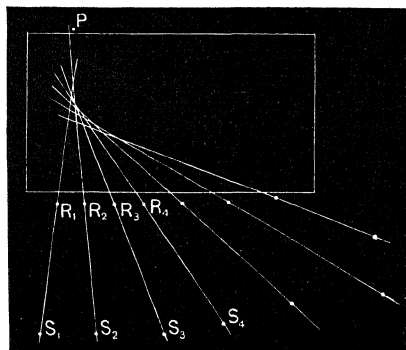


FIG. 17.—Caustic by Refraction at a Plane Surface.

pin, place two pins to appear in a line with it, one, R_1 , close to the glass, and the other, S_1 , at a distance of about 10 cms. In the same way place a series of pins, $R_2, R_3, R_4, R_5, \dots$ along the front edge of the block, with another series of pins, $S_2, S_3, S_4, S_5, \dots$ each at about a distance of 10 cms., so that the pins appear in pairs to be in a line with the one seen through the block. Remove the block and join each pair by a line, producing each line until it meets its neighbouring one. Draw a curve to touch all these lines and having its apex at the apparent position of the first pin when seen normally through the glass. This curve is the *caustic*, or rather is one half of it, the other half being on the other side of the normal line OP, and being similar.

5. Reflection at a Curved Surface.

Apparatus.—A cylindrical mirror (the makers of the glass shades used to cover wax flowers, etc., will cut a ring from the bottom of such a shade for a few pence); one from a shade of about 6 inches in diameter will be most useful. It should be then cut in halves, making two semicircular rings; one of these must be silvered¹ on the inner and the other on the outer surface, the silver, of course, being protected by varnish. These rings are very fairly circular and give good results. Drawing-board; pins; and scale.

i. **Convex Surface.**—Place the convex semicircular piece of glass on the drawing-paper and run a pencil round its silvered

¹ It is sufficient to *black* the semicircles with some black varnish, one on the outer and one on the inner surface, as the images of the pins can be quite easily seen against the black, though of course not nearly so well as in a silvered mirror.

surface. Place a pin, P , about 2 inches from its surface, and looking in the mirror, insert pairs of pins, Q_1R_1 , Q_2R_2 , . . . to appear in a line with P as seen in the mirror. Remove the mirror, join the points Q_1R_1 , Q_2R_2 , and produce the lines until they meet. Draw a curve to touch all these lines. This curve is called the *caustic*.

The lines such as Q_4R_4 , Q_5R_5 , . . . which are near the pin P , will be found to meet one another nearly at the same point, S . This point is called the *image* of P . It is obvious, that S is only the image of P for light incident nearly normally on the surface, as the lines, such as Q_1R_1 , which are at some distance from P do not pass through S . In the determination of the formula giving the position of S , it is always assumed that the light is incident nearly normally, and this experiment shows the reason for that assumption.

The caustic will meet the circle at the point T where the tangent from P touches it.

ii. **Concave Surface.**—(a) Place the concave semicircular piece of glass on the paper and insert a pin, P , at about 1 inch from the concave surface. Run a pencil line round the surface of the mirror. See that the mirror is just touching this line and insert pairs of pins, Q_1R_1 , Q_2R_2 , . . . to appear in a line with the image of P . Then, as in the last experiment, it will be found that the image S is behind the mirror for rays which are incident near the centre of the mirror.

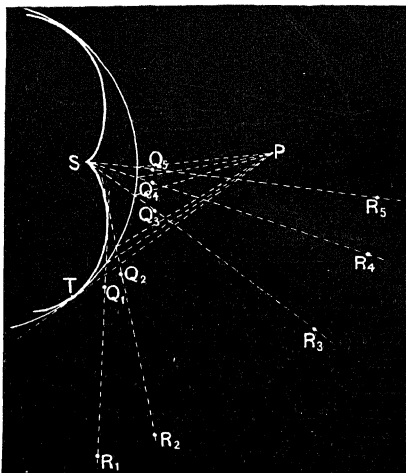


FIG. 18.—Caustic formed by a Convex Mirror.

Instead of inserting the pins $Q_1R_1, Q_2R_2, \dots$ it is just as easy, and a great deal quicker, to place a scale upon the paper with its edge pointing to the image of P as seen in the mirror, and

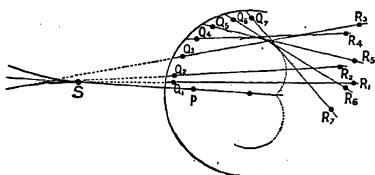


FIG. 19.—Directions of Rays from a point P after Reflection in a Concave Mirror.

to run a line along the edge of the scale, thus obtaining in one operation the same lines that will be produced finally by joining the pins Q_1R_1 , etc., and there is no risk of joining the wrong pairs of pins. Continue to draw these lines for rays in-

cident at points of the mirror further and further distant from the centre. It will be found that at a certain distance from the centre the image of the pin seems to broaden out and become indistinct; this is because the rays forming the image at this part of the mirror are, after reflection, almost parallel to one another, the image formed by those rays being at infinity.

Still further from the centre the lines will be found to meet on the near side. Remove the mirror and produce those lines which meet behind the mirror. Notice the shape of the caustic, which has two branches as in Fig. 19, and that the image is only formed at S when the light is incident nearly normally.

(β) Place P at a distance of about $2\frac{1}{2}$ inches from the same mirror, and repeat. The image S will now be formed on the other side of the centre of

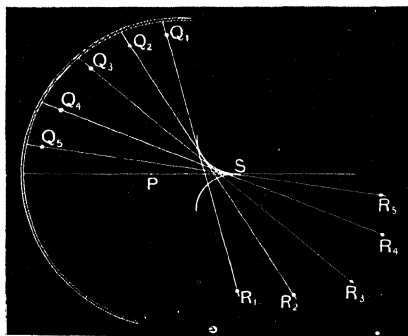


FIG. 20.—Rays from a pin P after Reflection in a Concave Mirror. They form a caustic of which the cusp S is the conjugate focus of P .

curvature of the surface of the mirror. Notice that in this case the point S is only the image for rays very nearly normal, and that those rays which are reflected from the extremities of the semicircular glass are a considerable distance from S. The distance of the reflected ray from S is called its *aberration*, and the distance measured along the line joining S to P—the axis—is called the *axial* or *longitudinal aberration* or the *spherical aberration* of the mirror.

(c) Place the pin P at about 6 inches from the surface, and repeat.

(d) Place P at exactly 3 inches¹ from the surface, that is, at the centre of the circle. In this case it will be found that the lines joining the pairs of pins, Q_1R_1 , Q_2R_2 , . . . always pass through P. There is therefore no aberration, and the image of P coincides with the pin itself.

iii. **Wave Surface.** *Apparatus.*—As before; also set square, tracing paper, and drawing pins.

Draw the rays as directed in Experiment 5,

ii. (a). (The drawing made in that experiment may be used.) Pin a piece of tracing paper over the drawing and, starting from a point on the central reflected ray about $\frac{1}{2}$ inch on the near surface of the mirror, draw a short line at right angles to that ray until it meets the next reflected ray, as in Fig.

21. Continue this line by a short line at right angles to the second

ray until it meets the third reflected ray. Again continue it by

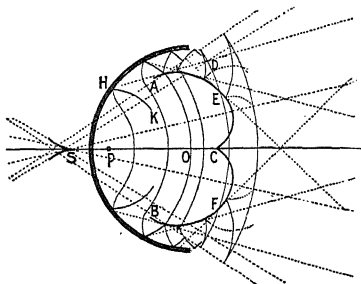


FIG. 21.—Five Wave Fronts of the Light reflected from a Concave Mirror. HK is a portion of the first wave from P that has not yet been reflected by the mirror; similar portions of the next three waves are shown, they are arcs of circles of which the centre is at T. The concave portion of the second wave near A becomes concentrated at the point on the caustic where the wave through O meets it; hence this third wave bends abruptly at the caustic. The part ECF of the caustic belongs to the other half of the mirror.

¹ If the diameter of the mirror is not 6 inches, these distances must be altered to suit it.

a line at right angles to this third ray until it meets the fourth ray, and so on, until the surface of the mirror is reached. Continue the line in a similar manner on the other side of the centre until it again meets the mirror. This line indicates the position and shape of the reflected wave, after it has travelled back a short distance from the mirror.

Beyond the point where this wave meets the mirror, the wave from the point P, not yet having reached the mirror, will still lie on a sphere whose centre is at P. Thus it may be continued a short way by describing arcs of circles with P as centre.

In the same way draw wave surfaces which pass through points on the central ray at distances of 1 inch, 2 inches, 3 inches, etc., from the mirror. Note that these latter reflected waves have a curious cusp as seen at D in Fig. 21.

6. Course of the Rays refracted through a Curved Surface:

i. **Convex Surface.**—*Apparatus.*—We shall require a glass ring cut from the bottom of a shade such as that already used as a cylindrical mirror. Cement this on a circular glass plate. A convenient diameter for this trough will be about 6 inches. Stand two or three pins upright in small discs of sheet lead. These can then be placed anywhere within the trough.

Place the trough upon the drawing-board and one pin within it about 2 inches from the edge. Pour water in to

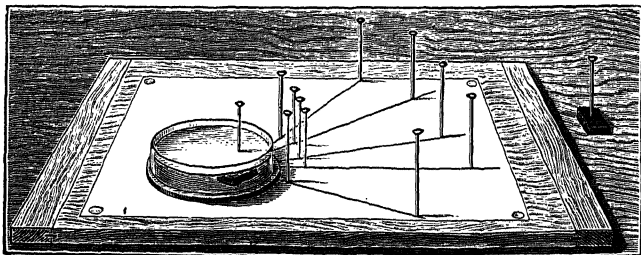


FIG. 22.—Refraction at a Curved Surface.

the depth of about half an inch, and looking through from without, place pairs of pins, one of each pair near the trough and the other about 10 cms. away, so that they seem in a

line with the one inside. We now want to mark the outline of the trough. If the glass plate does not project beyond the ring, this is quite easy. But otherwise, knowing its diameter, a circle may be described with compasses set to the right size passing through two points on its circumference (determined by actual measurement). Each pair of pins must now be connected by a line, and the lines produced until they meet. The caustic may then be drawn touching all these lines. The geometrical image of the pin is the apex of this caustic.

Measure the distance of the actual pin within the trough from the surface, and that of its image from the surface, also the radius of the trough. Make a drawing on a reduced scale in your notebook, and enter the measurements upon it.

ii. **The Course of Parallel Rays, represented by Pairs of Pins, through this Trough, and the Principal Focus.**—It will easily be seen that only those rays which pass normally through the surfaces go at all accurately through the geometrical focus, and that there is a considerable aberration for rays that are far from the central line.

7. Paths of Rays refracted through a Cylindrical Lens.

Apparatus.—The lens may be made by cementing strips 3 or 4 inches long, cut from rings similar to those already used but of a diameter of 8 or 10 inches, on suitable bases. In this way troughs to represent a plano-convex or concave, double convex, meniscus or double concave lens may easily be formed. Instead of these troughs, a piece of a cylindrical glass lens may be used. A strip a quarter of an inch deep, cut from a cylindrical lens of about 10 inches focus and $2\frac{1}{2}$ inches diameter, is very suitable. Half a cracked lantern-condenser may be used.

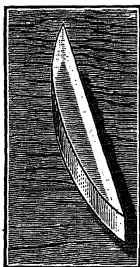


FIG. 23.
Cylindrical Lens.

Stand the lens upon a piece of ruled foolscap so that its plane may be perpendicular to the ruling. Looking through the lens, place pairs of pins, one close to the lens and one at about 10 cms. distance, to appear in a line with each line on the foolscap, as seen through the lens. Draw the trace of the

surfaces of the lens on the paper. Move the lens and connect up the pins. The caustic may now be drawn in, to touch these lines. In the case of a plano-convex lens, the rays should be traced through in both directions, and the distance of the geometrical focus from the surfaces measured, and also the

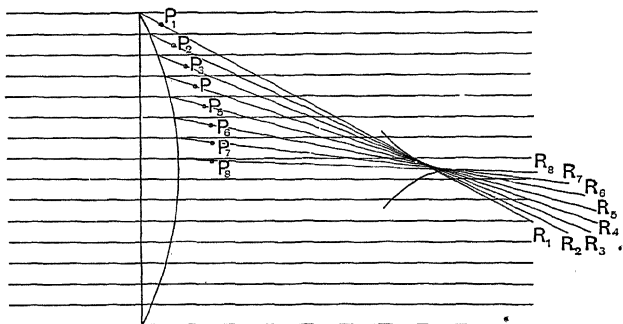


FIG. 24.—Refraction through a Lens.

difference in shape of the caustic observed. It will be seen that the aberration is very much greater,—as indicated by the angle of the caustic being more obtuse,—when the parallel rays enter the lens by its plane surface than when they enter by its curved surface.

8. Formation of the Rainbow.

Apparatus as in Section 6 (i.), except that about half the circle is to be silvered on the outside.¹

i. **Primary Rainbow.**—Place a pin, P_1 , within the trough close to the silvered surface. Looking through the trough obliquely, place two pins, R_1 , S_1 , to appear in a line with P_1 , and two more pins, T_1 and V_1 , to appear in a line with the image of P_1 and S_1 , formed by reflection in the mirror. Place two other pins, S_2 and R_2 , in such positions that the line joining them

¹ If the trough is not silvered it *must not* be blacked, as the chief reflection will occur at the outer glass-air surface. The pin itself is very difficult to see, but if a silvered bead be threaded on it, in a good light, the bright spot formed by reflection in this bead can be seen against a black background, such as a roll of black velvet, and the experiment may be performed as first described.

is parallel to the line joining S_1 , R_1 , and looking along this line, place P_2 within the trough to seem in the same line. Again, looking in the drop from the other side, place T_2, V_2 , to appear in the line with the image of P_2 , S_2 , R_2 , reflected from the silvered surface. Proceed in this way to find the direction

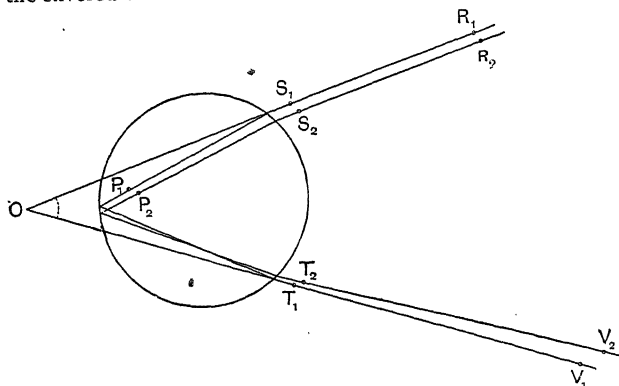


FIG. 25.—Primary Rainbow.

both within and without the trough of a series of rays all parallel to R_1S_1 .

It will be found that one ray, $R_1S_1T_1V_1$, is less deviated than those on either side of it. It is in this direction that the raindrop would appear brightest if illuminated by a beam of light parallel to RS . Produce the lines R_1S_1 , and V_1T_1 , to meet at O , and measure the angle R_1OT_1 . It should be $41^\circ 32'$.

ii. **Alternative Method with an Unsilvered Trough.**—If we assume that the reflection follows the ordinary laws of reflection, a silvered trough is not necessary. Place the pin P in the trough touching the far surface. Looking through the trough obliquely, place pairs of pins R_1S_1 , R_2S_2 , etc., to appear in a line with P . Or, which is much quicker, place a scale on the paper to appear in a line with P , and run a pencil along its edge. When the scale is almost tangential to the trough so that the light enters the surface very nearly at grazing incidence, it will be seen that the image of the pin broadens out. It is this position which gives the minimum deviation. On moving

the eye slightly this broad image will separate into two—one moving inwards towards the pin, and the other outwards to become the tangent to the surface. Follow the latter and draw the ray. Do this on each side.

It will be found that the rays obtained from the second images of the pins, namely those that appear to move towards the tangent, cross the others. The total deviation of these is therefore greater.

When the image broadened out, the rays became practically parallel. Produce any pair of these parallel rays, one on each side, until they meet, and measure the angle. It should be twice $41^{\circ} 32'$ (*i.e.* 83° about).

The angle seldom comes quite right as the trough is not sufficiently circular—a very small deviation from the circle having a very marked effect when the rays strike the surface at such oblique angles.

iii. **Secondary Rainbow.**—The only difference between this experiment and the last will be the selection of the direction of RS, which must slope as shown. The vessel must be so placed that the two reflections occur upon the silvered part of the trough.

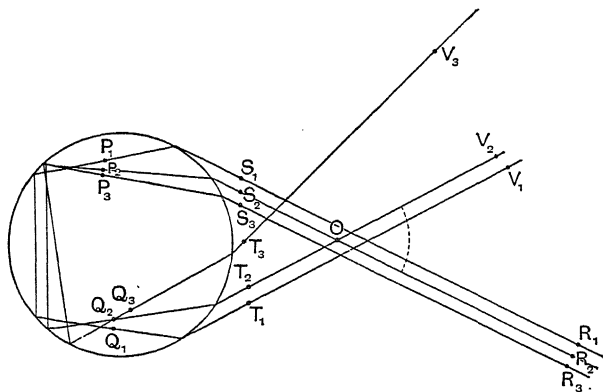


FIG. 26.—Secondary Rainbow.

A series of ruled lines such as those on a piece of ruled foolscap paper may be placed without the trough, in place of the parallel pairs of pins R, S. By looking at this system in

the direction VT, it is easy to determine the one which is least deviated, and to see that the deviation of the one on each side of this is very nearly the same as that of this one, but that the deviation increases very rapidly for those still further away.

Having found the one, $R_2S_2P_2Q_2V_2$, which is least deviated, measure the angle R_2OV_2 . It should be $51^\circ 58'$.

iv. **Artificial Rainbow.**—*Apparatus.*—An aspirator ; glass and rubber tube ; a short piece of glass tube with a bore 1 mm. in diameter ; drawing-board and pins ; gas flame—preferably incandescent—in a box with a small aperture, or a simple candle ; blocks and beaker.

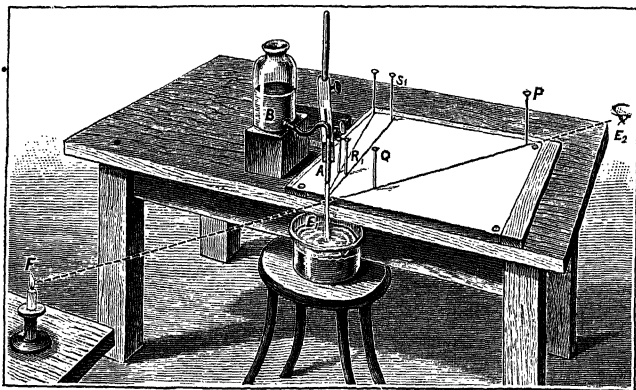


FIG. 27.—Arrangement of Apparatus to measure the Angles of the Primary and Secondary Rainbows. Standing back to the candle F, with the eye at E, the primary bow is seen in the direction R_1S_1 . The eye is at E_2 when inserting the pins P, Q, in line with F and the jet A.

Attach the millimetre tube to the tubules of the aspirator, and support it vertically in the clip so that a vertical stream of water may flow out, which can be received in a beaker. Prop up the drawing-board so that its upper surface is on a level with this stream. Place the gas flame at a distance of 3 or 4 feet from the jet, turning the box so that the light may fall upon the jet. Then, on bringing the eye as close to the jet as possible without intercepting the light, and looking through the jet, with the back

to the light, the primary and secondary rainbows will easily be seen, each accompanied by a large number (15 or 20) of supernumerary bows. The primary rainbow is the one nearest to the line joining the flame to the jet.

Insert in the drawing-board a pair of pins, one close to the jet and one as far away as possible in a line with the jet and the primary rainbow. Do the same for the secondary rainbow. Lastly, insert a pair of pins in the continuation of the line joining the light to the jet.

To do this go round to the far side of the board, and with the eye nearly in the plane of the board, look along the line joining the jet to the candle. In this position the surface of the paper or drawing-board will appear slightly shining; in fact, it will partially reflect the light of the candle, and a shadow of the jet will be seen along the surface. Insert one pin close to the jet in this shadow, and another at as great a distance as the board will allow.

Join the pins in pairs, and measure the angle between the lines so obtained. (As the lines will of course meet beyond the edge of the drawing-board, it will be found necessary to draw parallel lines on the board in order to measure these angles.)

Considering the simple nature of this experiment it gives remarkably accurate results. If the pins inserted in the direction of the rainbow are placed to appear in a line with the yellow of that bow, the angle given by c should be $41^{\circ} 32'$, and the result can easily be obtained to half a degree. The difference between the deviation of the red, yellow, and violet, can easily be measured, amounting as it does to about two degrees.

9. Determination of the Refractive Index of a Liquid by the Critical Angle.

When a ray of light travelling in a denser medium strikes the surface separating it from a rarer medium, it generally is partly reflected and partly refracted into the rarer medium, where it makes an increased angle with the normal. For a certain angle of incidence this angle, the angle of refraction, becomes 90° . If the angle of incidence be still further increased, there will be no refracted ray: the whole light will be reflected, and we obtain what is known as total reflection. The largest angle of incidence for which any of the light passes into

the rarer medium is called the critical angle; and since the angle of refraction is then 90° , and the light is passing from the rarer into the denser medium—

$$\frac{1}{\mu} = \frac{\sin i'}{\sin 90^\circ} = \frac{\sin i'}{1};$$

and therefore—

$$\sin i' = \frac{1}{\mu}.$$

If a parallel plate of some other transparent substance separates the two media, when the ray reaches the critical angle from that substance to the rarer medium, the angle of incidence on the plate from the denser medium will be the critical angle for that medium and the rarer one; for instance, if a glass plate separates water from air as in Fig. 28, and if the plate be rotated until the ray, PQR, strikes the surface, R, at the critical angle, i' , for glass and air, then it strikes the surface at Q at the critical angle, i , of water and air.

For, if μ be the refractive index from water to air, and μ' the refractive index from glass to air, the refractive index from water to glass will be $\frac{\mu}{\mu'}$, and—

$$\frac{\sin i'}{\sin i} = \frac{\mu}{\mu'}$$

Thus, if $\sin i' = \frac{1}{\mu}$, we shall also have $\sin i = \frac{1}{\mu'}$. The normal to the glass plate, therefore, makes an angle equal to the critical angle with

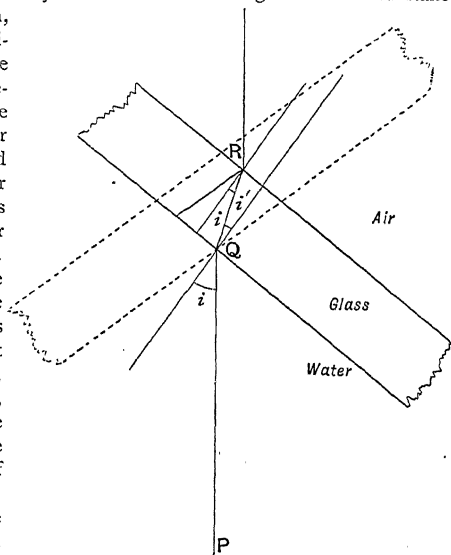


FIG. 28.—Critical Angle.

the incident ray. Therefore, if we set the glass normal to the ray, and rotate it until the critical angle is reached, and measure the angle through which it is turned, the reciprocal of the sine of this angle will be the refractive index of the water.

Instead of attempting to set the glass at right angles to the ray, it will be easier to turn it into the position shown by the dotted lines, in which the ray is again incident at the critical angle. The angle between the two positions of the glass will now be double the critical angle.

Apparatus.—A small glass tank about 2 or 3 inches cube—preferably with parallel sides, though this is not very important ; drawing-board ; pins ; millimetre scale ; two pieces of thin patent plate about 2 inches square ; tinfoil ; shellac varnish ; and a wooden stool about 6 inches long, tall enough to put over the trough.

Cut a piece of tinfoil the size of the glass plates and remove the greater part of the centre, leaving a narrow strip all round.

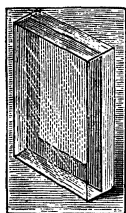


FIG. 29.—Glass Plates enclosing a layer of Air.

Clean the two pieces of glass, and placing this tinfoil mask between them (Fig. 29), attach them together top and bottom with cotton, and shellac varnish the edges so as to make them air-tight. A layer of air will thus be enclosed between the two glass plates—the thinner this layer of air the better. Cut a slot in the wooden stool parallel to its long side, as in Fig. 30, and insert the upper end of the double plate above described, fixing it in position with wedges and the shellac varnish. To each foot of the stand attach a needle to act as a pointer.

Place the drawing-board in front of a window, and put the trough on the centre of the drawing-board. Nearly fill it with water. Insert two pins, P and Q, perpendicular to the face of the trough. Place the stool over the trough so that the double glass plate containing the air-film may dip into the water. Then, looking through the trough in the direction PQ, the pin Q should be visible through the water and the glass plates. Now turn the stool round, and in a certain position it will be found that the pin Q will disappear, and one half of the field will become dark. Mark the positions of the needles on the paper as at A, B, when the blue colour, with which the dark part of the field terminates, just reaches Q. Now turn the

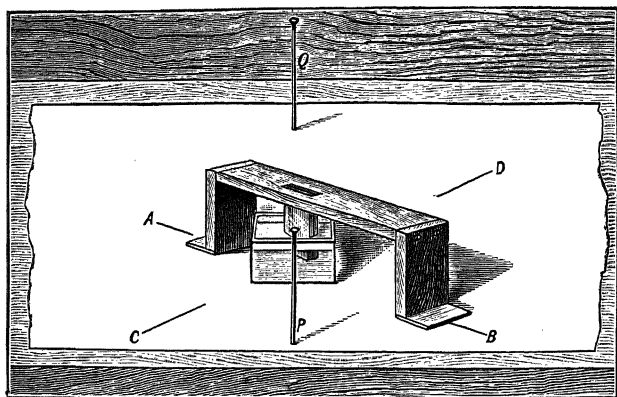


FIG. 30.—Refractive Index of Water by the Critical Angle.

stool round until the pin Q is again on the point of disappearing, and mark the lines C, D, on the paper.

The ray after passing through the first glass plate and the air-film will, until the critical angle is reached, enter the second glass plate, and will emerge from it into the water parallel to the original incident ray, so that the light coming from Q passes right through to the eye at P. But when the critical angle is reached, the ray on entering the air-film passes along at grazing incidence, and strikes the tinfoil instead of the second glass plate, and no light from Q will reach the eye. If the glass plates are separated by a larger space, the light from Q will be cut off before the critical angle is reached.

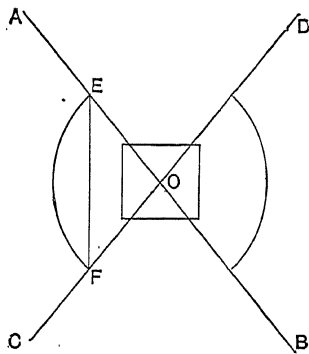


FIG. 31.—Measurement of Critical Angle.

The blue colour indicates the disappearance of the yellow, and shows that we have reached the critical angle for yellow light; the refractive index obtained should therefore be the refractive index of water for yellow light.

Now remove the stool and trough, join the lines AB, CD (Fig. 31), meeting at O. With centre O, draw a circle, cutting these lines at E and F. Join E, F. The critical angle will be half the angle EOF.

And therefore—

$$\sin i = \frac{\frac{1}{2}EF}{OE};$$

or—

$$\mu = \frac{2OE}{EF}.$$

10. Determination of the Refractive Index of a Liquid by the Critical Angle from the Liquid to Glass.

Apparatus.—A cubical glass block or a right-angled glass prism, the refractive index of which is known; drawing-board, pins, protractor; a piece of ground glass or paper screen that can be set up vertically; gas flame—preferably incandescent.

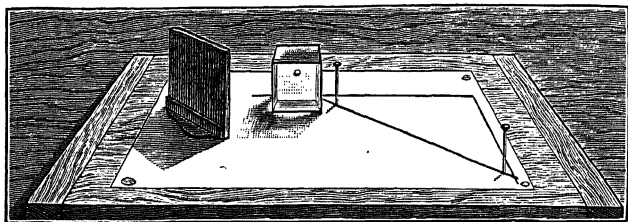


FIG. 32.—Refractive Index of a Drop of Liquid.

Draw a long line AB on the drawing-board. Put a drop of the liquid, the refractive index of which is to be found, on one face of the block, and place the block on the drawing-board with this face, CD, vertically over the line AB. Look in the direction PQ through the face DE of the block, and place the ground glass, or paper screen, S (which must be well illuminated),

so that its image is seen by reflection in the face CD. At the point where the drop of liquid is placed on the face CD, the reflection will generally be less perfect; on moving the eye towards B, at a certain angle, the liquid will begin to disappear, and the reflection become total. When the drop has only partly disappeared, a vertical blue line will be visible. Insert two pins, P and Q, on the line joining the eye to this blue line. In this direction, the yellow light has just reached its critical angle.

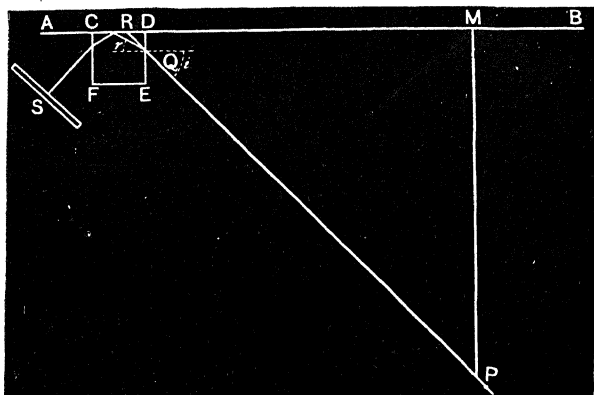


FIG. 33.—Refractive Index of a Drop of Liquid.

Thus, calling i and r the angles of incidence and refraction at the surface DE, since the ray is incident at CD at the critical angle, we have—

$$\cos r = \frac{n}{N};$$

n being the refractive index of the liquid, and N that of the glass; also—

$$\frac{\sin i}{\sin r} = N,$$

therefore—

$$\sin r = \frac{\sin i}{N}.$$

Square and add, we get—

$$\sin^2 r + \cos^2 r = 1 = \frac{n^2}{N^2} + \frac{\sin^2 i}{N^2};$$

Therefore—

$$n = \sqrt{N^2 - \sin^2 i} \dots \dots \dots (i)$$

Produce PQ to meet AB at R, and drop a perpendicular, PM, on AB. Then—

$$\sin i = \frac{PM}{PR}.$$

Thus, as N is known, n can be found by substituting in equation (i).

11. Determination of the Refractive Index of a Substance by Measuring the Apparent Thickness.

i. **Liquid.**—*Apparatus.*—A shallow vessel will be required to contain the liquid, preferably of glass, and a microscope with a low power objective; it should have a scale on the coarse adjustment.

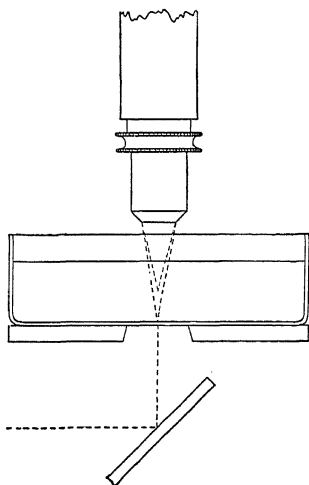


FIG. 34.—Refractive Index of a Liquid.

The microscope is set up with its tube vertical, and is focussed upon the bottom of the empty vessel. The reading on the scale attached to the coarse adjustment is observed. [In default of such a scale, the height of some definite point on the tube of the instrument can be read with an ordinary millimetre scale.] Without moving the vessel the liquid is poured in to a depth of 1 or 2 cms., depending upon the focal length of the objective, and the microscope is focussed upon the

image of the bottom as seen through the liquid, and its position again noted. Lastly, it is to be focussed upon the surface of the liquid; for this purpose a little lycopodium powder may

be scattered upon the surface. The refractive index is the ratio $\frac{\text{the real depth}}{\text{the apparent depth}}$.

ii. **A Glass Block.**—*Apparatus.*—A cubical block of glass about $1\frac{1}{2}$ inch edge ; a microscope with a low power objective, its body either horizontal or vertical—preferably the former ; blocks and millimetre scale.

If the block is new and free from scratches, it will be necessary to make its surface visible, either by fingering it, or by attaching a small piece of stamp-paper. Set it up so that the stamp-paper can be seen in the microscope after refraction through the block, and focus the microscope upon it. If there is a scale attached to the rack of the microscope, take the reading upon it ; if not, measure to some fixed point on the tube with the millimetre scale. Withdraw the microscope until the front surface of the block is in focus, and measure the amount the objective has been moved, again using either the scale on the instrument, if it has one, or the millimetre scale. The amount of the motion is the apparent thickness of the block ; its real thickness can be determined directly with the millimetre scale. The refractive index is the ratio $\frac{\text{the real thickness}}{\text{the apparent thickness}}$.

iii. **A Micro Cover Glass.**—*Apparatus.*—Some pieces of micro cover glass ; a microscope, of which the milled head of the fine adjustment must be graduated. The actual value of the gradations will not be required. A higher power (at least $\frac{1}{2}$ inch) objective will be necessary.

Place the cover glass, whose surface must be rendered visible by some means (it is sufficient to finger it), on the stage of the microscope, so as to observe a part of its surface near the edge—the edge may be in the centre of the field. By focussing first the lower and then the upper surface, determine its *apparent* thickness.

Fix the attention now upon the edge itself ; the terminations of the upper and lower surfaces will be visible owing to the

large cone of rays which the objective admits, and therefore, the *actual* thickness of the surface may be measured, using the same fine adjustment screw. The ratio of the real to the apparent thickness will be the refractive index.

As we are dealing only with the *ratio* of the real and apparent thicknesses, it is sufficient to measure them in divisions of the milled head, it is not necessary to determine their absolute values in centimetres.

12. Selection of a Piece of Parallel Glass, and Rough Determination of the Angle between the Surfaces of a Piece of nearly Parallel Glass.

If a distant bright point be observed at very oblique incidence in a piece of parallel unsilvered or silvered glass, images are formed by reflection both at the upper and

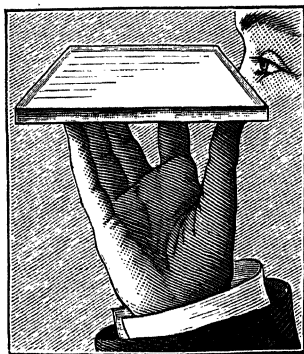


FIG. 35.—Examination for Parallelism.

lower surfaces, and also after three, five, or more internal reflections. If the surfaces of the glass are truly parallel, these images will all approximately coincide, and for an object at an infinite distance, will do so absolutely, for the rays after reflection will all be parallel, and will therefore appear to come from the same point. If a piece of ordinary glass be held up so that a distant flame is observed at almost grazing incidence, a series of images of the flame will nearly always be visible which gradually diminish in brightness. If the glass

be now rotated in its own plane, this series will be seen to rotate also. The series is due to the two surfaces not being truly parallel.

Let S (Fig. 37) be the distant bright point and AB and CD be the upper and lower surfaces of the mirror, and, to simplify the figure, we will neglect the refraction so that we may treat AB and CD as two ordinary mirrors. Rays from S which enter the eye, after reflection at these mirrors (since the lines SP , SQ are parallel, and the surfaces AB and CD are also parallel), must obviously either coincide or be parallel. Thus, as S is distant, only a single image will result.

In Fig. 38, let the surfaces AB and CD make a small angle with one another, but suppose them each to be perpendicular to the plane of the paper so that their line of intersection is normal to the paper.

Let R_1 , R_2 , be the images of the source formed by the

surfaces AB and CD respectively. The angle S_1PR_1 is twice the angle

FIG. 36.—Series of Images of a Distant Candle seen by Oblique Incidence in a nearly Parallel Mirror. (This figure should be looked at obliquely.)

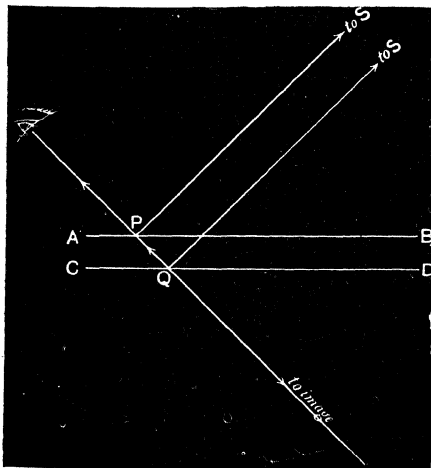
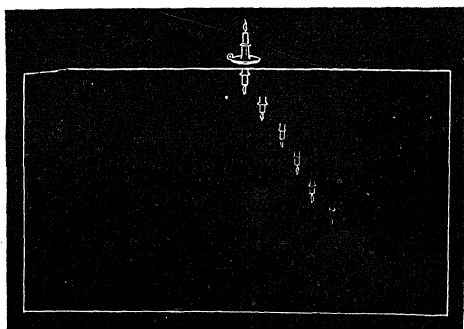


FIG. 37.—Single Image of a Distant Flame formed by Two Parallel Mirrors.

S_1PB , the angle S_2QR_2 is twice the angle S_2QD ; therefore, the angle R_1ER_2 must be twice the angle between AB and CD. It can easily be shown that, if R_3 is the image formed after three reflections, the angle R_3ER_2 is also equal to twice the angle between the mirrors and so on. A series of images of the distant object will thus be formed at equal angular intervals.

If the glass be

in its own plane, so that the line of intersection of the surfaces is no longer perpendicular to the plane of incidence, the images formed by

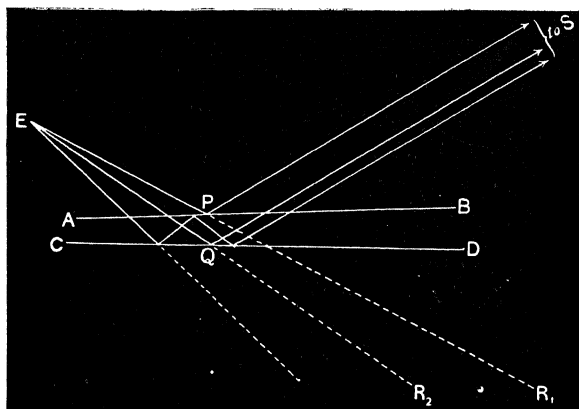


FIG. 38.—Multiple Images of a Distant Flame, formed by Two Mirrors not quite parallel to one another.

successive reflections will no longer lie in the plane of the paper. They will appear to tail-off either to the right or the left.

Apparatus.—Pieces of plate-glass, candles, and a metre scale.

i. **Examination and Selection.**—Light one candle and put it as far away as the size of the room will permit. Examine the appearance of the image or images, in one of the pieces of glass held very obliquely, so that the candle seems only a short distance from its image formed in the mirror. If a series of images is formed, rotate the glass and observe the rotation of the images. They make one complete révolution by the time the plate has been rotated once in its own plane. Also move the plate in its own plane so that the reflections occur at different parts of its surface. If the surfaces are not perfectly plane (so that the angle between them varies from point to point), this will be shown by the images closing up or opening out. If only a single image is visible or if the series of images be nearly coincident, the plate is either parallel or nearly so. In the latter case it

may happen that some part of the plate may be found to be parallel. Should such a part be found, it should be marked and put aside, as it will be valuable for other experiments.

ii. **The Angle between the Surfaces.**—Support the mirror with its plane vertical. Rotate it in its own plane until the images of the distant candle are in a horizontal line. Then arrange two other candles at about the same distance from the observer as

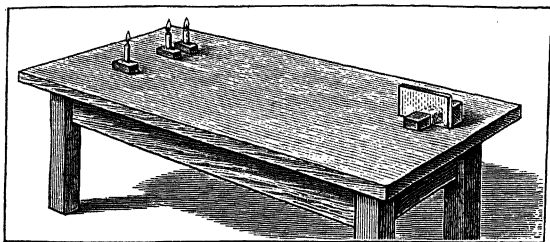


FIG. 39.—Angle between Surfaces.

the first, to appear (as seen through the glass if unsilvered, or seen just over the edge of the glass if silvered) to coincide with the first two images of the first candle. Measure the distance apart of these last two candles (a) and the distance from them to the eye (d). Then the angle between the two surfaces will be given approximately by $\frac{a}{2d}$, in circular measure.

ADDITIONAL PRACTICAL EXERCISES TO CHAPTER I

1. Stand two mirrors on a paper so that their surfaces are both vertical and at right angles to one another. Stick a pin in the paper about 3 inches in front of one mirror and 1 inch in front of the other mirror. Follow with pins the course of two rays from this pin to an eye placed to observe the image which has been formed after two reflections in the mirrors, producing the rays in each case until they meet, and so locate the positions of the successive images. See that the images lie on a circle whose centre is at the intersection of the mirrors, and passes through the original pin.

2. Reduce the angle between the two mirrors in the last experiment to 60° . Note the number of images formed, and see that when the

angle is not exactly correct, the diagonally opposite image of the pin seems suddenly to alter its position as the head is moved sideways, and that this alteration occurs when the line joining the image to the eye passes from the one mirror to the other. Follow the path of two rays which are forming this image from the pin to the eye. Again see that the successive images lie on a circle.

3. Place two mirrors facing and parallel to one another, and about 2 inches apart. Insert a pin between the mirrors and about half an inch from one of them. Looking in the mirrors a little to one side, note that a series of pairs of images are produced by these successive reflections. By the method 1 (iii.) find the positions of these successive images.

4. Trace a ray in the last experiment from the pin to the eye which has suffered three reflections at the parallel mirrors.

5. In Fig. 11 the two pins seen through the prism do not appear close to the surface of the prism, and do not reach down to the lines AB, AC. Why is this? Examine which is the pin marked P in the figure, and trace the rays by which the pin is seen in this position.

6. From the measurements in Experiment 3 (i.), calculate the refractive index of the prism from the formula—

$$\mu = \frac{\sin \frac{1}{2}(\delta + A)}{\sin \frac{1}{2}A},$$

δ being the angle FOS, and A the angle BAC of the prism (Fig. 12). The sines of the angles are to be taken from the table of natural sines given at the end of the book.

7. In Experiment 3 (ii.) draw a series of pairs of lines RP, SQ. See that (if the prism is isosceles, for instance, an equilateral one) the lines RP and SQ make equal angles with BC, and thus the prism acts in a sense as a mirror. Produce the pairs of lines to meet. They do not meet in the same plane. Note that the pin S may be placed on the other side of the line BC produced, and the reflections will still take place.

8. From the measurements made in Experiment 6 (i.), calculate the refractive index by means of the formula—

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r},$$

or—

$$\mu = \frac{\frac{r}{u} - 1}{\frac{r}{v} - 1},$$

in which u , v , and r are the distances of the pin, image, and the centre respectively, from the surface of the trough at which refraction takes place when the incidence is nearly normal, and are to be reckoned positive when they are measured towards the incident light. As this is the case here, they will all be positive in the above formula.

9. In Experiment 6 (iii.) calculate the geometrical focus, and compare its position with that found by experiment (assuming the refractive index of water to be 1.33) by making two applications of the formula—

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}.$$

10. Place the concave semicircular glass on a sheet of paper as in Experiment 5 (ii.). Light a wax match and hold it 1 inch, $2\frac{1}{2}$ inches, 3 inches, and 6 inches from the surface. Note that the light reflected from the concave surface is much brighter along the caustic. Note that the paper is a little brighter within the caustic than it is outside. Hold a lead-pencil against the inner surface. It will form a shadow on the paper, which comes to a point on the caustic. Move the pencil along the surface. See that the point of the shadow always lies on the caustic, and the shadow is always tangential to the caustic. The shadow, of course, marks the direction in which the light it cuts off would have been reflected, and therefore shows that the reflected light passes through the caustic. As the shadow is pointed, it shows that the light reflected from the whole breadth of the pencil is concentrated into one point, and therefore explains the greater intensity there.

11. Instead of the stream of water of Experiment 8 (iii.), use a glass rod, and find the angles of the primary and secondary rainbow for glass. (The angle of the secondary bow will be found greater than a right angle.)

12. Place two right-angled prisms with their hypotenuses in contact to form a cube. On looking in one face of the cube only objects reflected in the hypotenuse will be visible. Insert a drop of water between the two faces in contact, the part wetted will become transparent.

13. Place this block on the drawing-board as in Experiment 10, and find the direction PR in which the block just ceases to be transparent. Measure the lines RM, PR, as in that experiment, and calculate μ_w from the equations—

$$\mu_w = \mu_g \sin \theta \quad \text{and} \quad \sin(\theta - 45) = \frac{1}{\mu_g} \cdot \frac{RM}{PR}.$$

CHAPTER II

MIRRORS AND LENSES

13. Position and Nature of the Image formed by a Concave Lens.

Apparatus.—Concave lens of about 15 cm. focal length ; stand to hold the lens vertical ; a card with large printed letters upon it, also supported in a vertical plane.



FIG. 40.—Concave Lens.

Place the lens at a distance of 5, 10, 15, 20, . . . cms. from the card, and examine the image seen through the lens. Move the eye to the left, and, treating the lens as a window (p. 5), determine whether the image is in front of or behind it. Observe also the nature and character of the image—whether it is erect or inverted, magnified or diminished—at each distance.

Enter the results thus—

Distance of object from lens.	Is the image on the same side of the lens as the object?	Is it erect or inverted?	Is it magnified or diminished?	Is it nearer the lens than the object?

To make the entries in the last column, we must remember, as pointed out above, that when viewing objects at different distances from the observer that are practically in a line with one another, a movement of the observer to right or left will cause the *farther object to appear to move relatively to the nearer one in the same direction as the observer*: i.e., if he moves to the left, the farther object will move to the left relatively to the nearer one. They may both move to the left or right, but in either case the further one will finally be to the left of the nearer one. The object as seen outside the lens, and its image as seen through the lens, will be found both to move to the left across the lens if the observer moves to the left; but the motion of one of them across the lens will be greater than that of the other. The object is therefore moving relatively to its image. If this relative motion is to the left, the object is further off than its image.

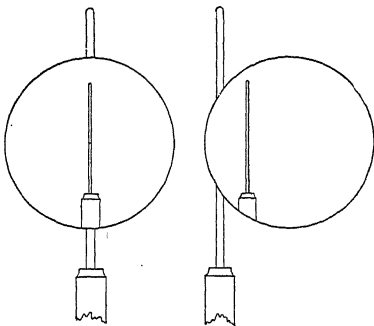


FIG. 41.

FIG. 42.

Images in a Concave Lens.

FIG. 41.—Central position when a knitting needle is used as an object.

FIG. 42.—If it appears as in this figure, since the observer has moved to the left, and the object is now to the left of the image, the object must be further away than the image.

found both to move to the left across the lens if the observer moves to the left; but the motion of one of them across the lens will be greater than that of the other. The object is therefore moving relatively to its image. If this relative motion is to the left, the object is further off than its image.

14. The Position and Nature of the Image of an Object formed by Refraction through a Convex Lens.

Apparatus.—A convex lens of about 15 cms. focal length; stand and printed card as before; small gas flame and screen; knitting needle in a vertical stand.

i. Place the lens at a distance of 5, 10, 15, . . . cms. from the card, and standing at a distance of 2 or 3 metres from the lens observe the image as seen through the lens.

Enter the results in columns as in Experiment 13.

It will be found that at some distances the image is erect and

at other distances it is inverted. Also at some distances it will be magnified, whilst at other distances it will appear diminished : it is also sometimes on one side of the lens, and sometimes on the other. At one distance the image will be indistinct. This distance should be noticed.

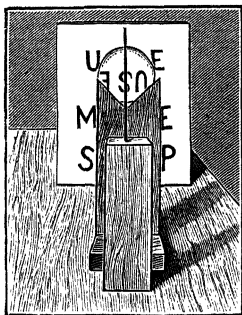


FIG. 43.

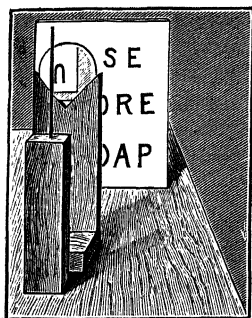


FIG. 44.

In Fig. 44 the observer has moved to the right, and the needle has moved from the S to the U, or has moved to the right relatively to the lettering. Thus, the needle is farther off than the image, and must be moved towards the observer.

At those distances at which the image is formed on the observer's side of the lens the image is real and can be received upon a screen ; but there would not be sufficient light from the card to see this image. To find the image, place the knitting needle between the observer and the lens. On moving to the left, both image and knitting needle, being on the near side of the lens, will appear to move across its surface towards the right. If the knitting needle is exactly in the same position as the image, it will move at exactly the same rate as the image across the lens, and will not seem to move relatively to the image itself. But this is not likely to occur at the first trial. The knitting needle will either be some distance in front or behind the image. If the knitting needle is farther from the observer than the image, when he moves to the left it will seem to move across the image towards the left. As beginners usually find some difficulty in distinguishing the *relative* motion of the knitting needle across the image from the motion

of knitting needle and image across the lens, it is advisable to sketch the position of the knitting needle and the lettering, first when the observer is straight in front of the lens and again when he has moved some distance to the left. On comparing the two sketches it is easy to see if the needle has moved across the lettering to the left; if so, it is too far from the observer, and must be moved towards him. By repeating this several times, it is possible to find a position for the needle, in which this relative motion ceases, and then it and the image really coincide.

If we now replace the knitting needle by a screen and the printed card by a gas flame, a clear image of the gas flame should be formed upon the screen.

15. The Position and Nature of the Image formed by Reflection in a Convex Mirror.

Apparatus.—Convex mirror about 20 to 30 cms. radius of curvature, with stand, small gas flame, knitting needle in a vertical stand.

Set up the convex mirror in a vertical plane and place the gas flame at distances 5, 10, 15, . . . cms. from a card, and observe its image as seen in the mirror.

Enter the results in columns as before.

It will be found that the image is always formed behind the mirror. Place a knitting needle also behind the mirror; of course, only the upper and lower parts will be visible, the centre being obscured by the mirror itself. Looking at the image of the gas flame, place the knitting needle so that the gas flame seems in the same vertical line as the knitting needle. Move the head sideways to the right or left, and notice if the gas flame moves across the mirror at the same rate as the knitting needle; that is, whether it still seems in a line with the ends of the knitting needle even when the eye is considerably to one side. If not, consider whether the knitting needle is too far back or too

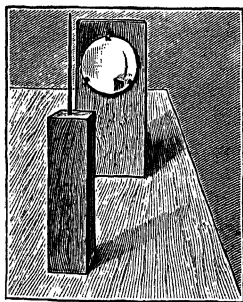


FIG. 45.—Convex Mirror.

near the mirror, using the rule given on p. 6, and move it accordingly. Do not move the needle by guess-work only and try to find its position by rule of thumb, but think first the direction in which it should move. If this is not systematically done, it will be found that not only will a great deal of time be wasted in obtaining even the approximate position, but the position will never be found with the same amount of accuracy as when the movement of the needle is made intelligently.

Observe whether the needle or the flame is nearer to the mirror.

16. The Position and Nature of the Image formed by Reflection at a Concave Mirror.

Apparatus.—The concave mirror of about 20 to 30 cms. radius of curvature, and of as large a diameter as possible (a *shaving-mirror* does excellently); stand, gas flame, knitting needle, and screen; black card, vase, and flowers.

i. **Real and Virtual Images.**—Proceed as described for the convex lens, placing the flame at 5, 10, 15, . . . cms. in front of the mirror, and observe it from a distance of 2 or 3 metres. Then, as in that case, the image will sometimes be on one side

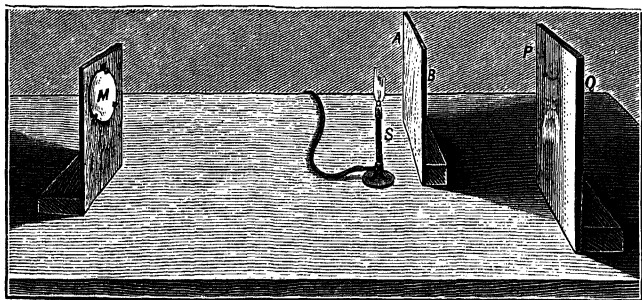


FIG. 46.—Real Image formed by a Concave Mirror.

of the mirror and sometimes on the other, and at some distance, will become indistinct. When the image is formed on the observer's side of the mirror, it will be real and can be received upon a screen. Find the position of the image by

setting up a knitting needle and using the parallax method as described for the convex lens, and finally replace the knitting needle by the screen. If the position has been correctly found, the image should appear well in focus. One difficulty will be found in this case; namely, that the direct light from the flame as well as that reflected from the mirror will both reach the screen. This can only be avoided by placing the flame and screen slightly on opposite sides of the axis so that the reflection from the mirror is slightly oblique, and then a card can be interposed to shade the direct light of the flame from the screen, as in Fig. 46, p. 42.

Notice that as the flame is brought nearer the mirror, the screen, in order to keep the image in focus, will have to be gradually withdrawn.

ii. **Centre of Curvature.**—Try to obtain a position in which the image is the same size as the object. It will be found that this occurs when the screen is the same distance from the mirror as the flame. This distance is the radius of curvature of the mirror, which may therefore be at once measured. If the flame is at a greater distance, the screen will have to be placed between the flame and the mirror (of course, it must be put slightly on one side of the axis and the flame slightly the other, or the light cannot reach the mirror, and therefore no image can be formed on the screen).

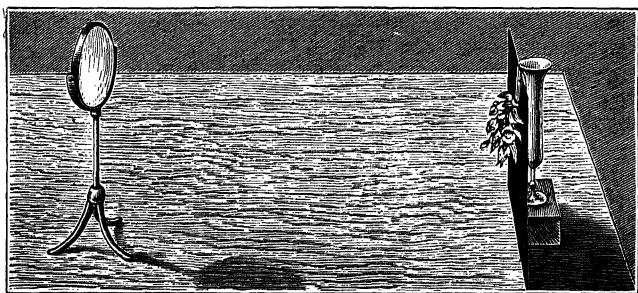


FIG. 47.—Formation of Aërial Image by a Concave Mirror.

iii. **Optical Illusion.**—Arrange a small flower vase on a block, so that its mouth may be on a level with, and near the centre of

curvature of, the mirror found above. Against this, between it and the mirror, stand a black card which also reaches up to the same level. Suspend a small bunch of flowers against the back of the card. Adjust the distance, and inclination, of the mirror until the image of the bunch of flowers is formed above the vase; and then on looking into the mirror from a little distance, it is difficult to realise that it is only the image that is seen.

ADDITIONAL EXERCISES ON CHAPTER II

Apparatus.—Some long focus concave and convex lenses (old spectacle lenses, which may be obtained from any opticians, of powers $\frac{1}{4}$ to 2 dioptries, will do excellently).

1. Hold a convex lens close to the eye, and move it up and down in a vertical plane. Look through at some moderately distant object. The image of this object will appear to move up and down also. Notice whether the movement is in the same direction as that of the lens; for instance, whether the image moves up when the lens is moved up.

Draw a diagram showing the course of rays from the object to the eye through the upper edge of the lens.

See that the displacement observed agrees with that which you would expect from your drawing.

2. Repeat this experiment with a concave lens. The motion of the image will be in the opposite direction to that caused by a convex lens. Make a drawing showing the course of rays through its upper edge, and compare the result of the experiment with your drawing.

3. Sort a series of lenses, separating the convex from the concave by this method. It will be found very difficult to distinguish a very weak convex lens from a concave lens by the appearance, but quite easy by this method.

4. Arrange the convex lenses in order according to their powers; that is, according to the amount of the displacement they produce. If two or more lenses produce equal displacements, they are of equal power, and should be put together.

5. Next arrange the concave lenses in a similar manner. Attempt to compare the convex and concave lenses; *i.e.* to find a convex and a concave lens which will, when placed in contact and moved up and down before the eye, produce no apparent displacement of the image. Such a pair will have equal focal lengths.

6. If in the fourth exercise there were two or more convex lenses of

equal power, and if there is a concave lens to pair with one of them, test it with the others, and see if it pairs with them also.

Hence test the equality of their powers. (If there is no concave lens that exactly compares, try with the one which is nearest.)

7. Suddenly place a piece of the plate glass used in Experiment 12 (p. 34) in front of the eye, or suddenly remove it, and see if there is any apparent displacement of the image. If there is, it shows that the two surfaces of the glass are not exactly parallel. Note the direction of the displacement, and hence determine in which direction the surfaces slope together. Compare the result with the appearance of the distant flame observed as in Experiment 12 ii.

CHAPTER III

FOCAL LENGTH OF MIRRORS AND LENSES—OPTICAL BENCH

17. The Optical Bench and Fittings.

A VERY simple form of optical bench can be made by attaching a metre scale to one side of the upper face of a board about

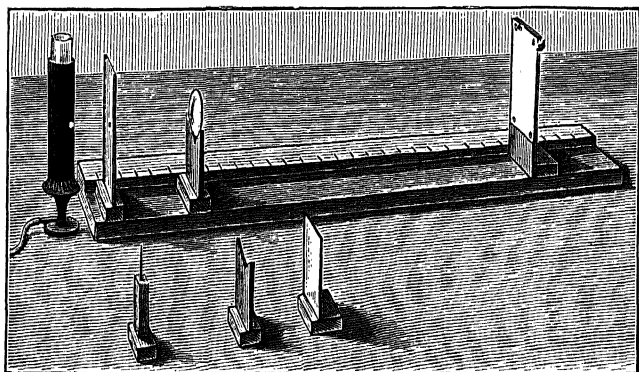


FIG. 48.—Simple Optical Bench.

5 inches broad and 1 inch thick, and supporting lenses on short blocks of wood in uprights cut with V slots, as indicated in the figure. The *object* can be a piece of thin board about 6 inches square, supported on another block, with a $\frac{1}{4}$ -inch hole exactly on a level with the centres of the lenses ; to the front surface of this attach a white card with drawing-pins, having a clean hole a little more than $\frac{1}{8}$ inch in diameter opposite the centre of

the larger hole in the wood, with a vertical and a horizontal cross-wire, for which a very fine glass capillary tube does excellently.

A lamp, or better an incandescent gas flame, is placed behind this. The receiving screen can be another similar board and card, but without any hole.

Owing to the warping of the wood caused by the heat of the lamp, it is better to make the first screen of zinc (the one with the hole in). This is bent into the shape shown in Fig. 52, and a card slipped in the turned over part, AB, so that there is an air space of about $\frac{1}{8}$ inch between the card and the zinc. The zinc, mounted on a wooden block as before, can be cemented in a groove cut in the wooden block (Fig. 49). The heights of the centres of the lenses and screens need not be more than about

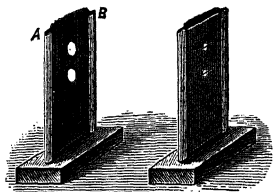


FIG. 49.—Cross-wire Screen with two holes. (When only one is required, the other is to be covered.)

3 inches. [It will be well to make them the same as that of the saccharimeter described p. 174, so that the parts may be interchangeable.] The blocks should be of the same width, so that when pressed against the metre scale, the centres shall be in a line. Each size lens will require a separate V block in order that the centre may remain at the same height; but as these are very easily made, and only about four lenses will be necessary, this is no serious matter.

AB shows the shape to which the zinc is bent, and the other figure shows the card in position.

Spectacle lenses may be used if desired. The most useful will be a 10-inch and a 20-inch convex lens, and a 14-inch and a 20-inch concave lens.

If special lenses are bought for it, 2 inches diameter will be found a convenient size.

A convex and a concave mirror, each of about 12 inches focus, will be wanted; also a plane mirror of good plate-glass, which must be mounted to stand vertically in a block.

One stand will be required to hold both convex lenses in contact, and another the 10-inch convex and the 14-inch concave

together. If an achromatic lens about 2 inches in diameter and 6 to 10 inches focal length be obtained (such as are used for marine glasses), the cost will probably be no more than that of a convex and concave lens. It will not only answer the purpose for which those lenses are wanted, but will be useful in other ways, and will be much better for the *auxiliary lens* in the methods which depend upon the use of a convergent beam.

A better form of bench, allowing the stands to be brought closer to one another and allowing the centres to be adjusted in line, is shown in Fig. 50 in plan.

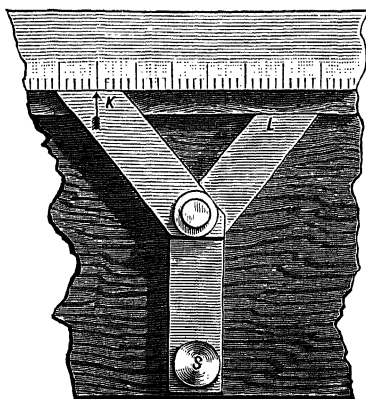


FIG. 50.—Plan.

Improved Optical Bench.

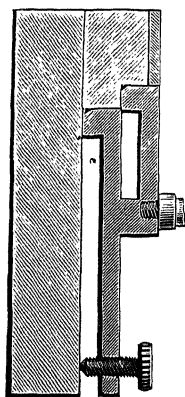


FIG. 51.—Section.

The section of the base shows a ledge, upon which one foot of the stand rests. A second foot rests on the base board, and the third foot is supported by a screw, S, also resting on the base. It will be noticed on examining the drawing of the stand itself, that this will allow the two stands to be brought very close to one another; the foot K of one of the stands that rests on the ledge, sliding above and across the foot L of the other that rests on the base-board. The screw S allows the stand to be tilted, and thus produces the effect of a transverse

motion, enabling the lens to be set nearer or further away from the edge of the bench. Each stand has a tube about 6 inches long, strengthened at the top with a brass cap, and is split and sprung inwards, to grip smoothly an inner sliding tube which carries the lens or other fitting.

For this bench it is preferable to mount the lenses and the various mirrors each on a separate inner tube, so that they may be interchanged readily, and so that they are less likely to be broken.

The back view of the screen next the lamp is shown, to illustrate the method of attaching it to the inner tube. ABCD

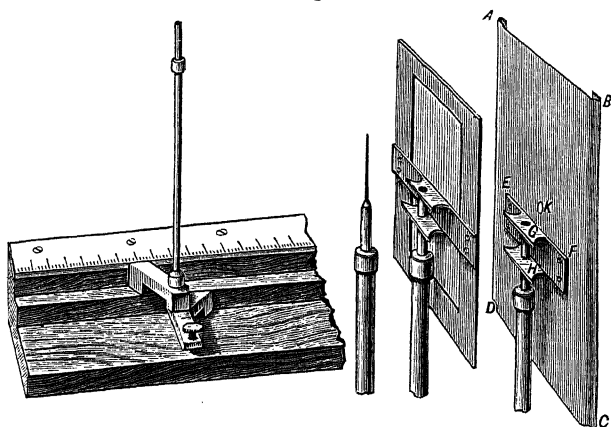


FIG. 52.—Optical Bench, Stand, Needle, and Screens.

(Fig. 52) is a piece of zinc bent as described before. To the back of this is screwed a thin brass plate, EF. Two brass plates, G, H, are silver-soldered to this, and a tube passes through a hole in each of these and is soldered in. K is the half-inch hole through the zinc and exactly opposite it is the small hole in the cardboard sheet.

The receiving screen is mounted on the inner tube in a similar manner, and consists merely of a hollow frame cut out of sheet brass, on which paper is stretched. A piece of good drawing paper is damped and left for about ten minutes to

stretch ; the front of the frame is then pasted and placed upon the damp paper ; some weights are put on, and it is left a day or two to dry. When dry it will be found to be stretched tightly and perfectly flat.



FIG. 53.—Lens Holder.

For holding any odd lenses that one may wish to experiment with, a very convenient stand can be made by joining a pair of curved pieces of brass into a cross-piece, fixed to the top of a brass rod that fits in the various stands. A nut with a conical upper surface screws up and down on this rod, and as it is screwed up, the edges of the curved pieces are forced together. These edges have a V cut on their inner edge, and therefore will clamp a lens of any size.

A steel rod with square ends about 8 or 10 inches long will be required for finding the index error (p. 54). This should be mounted at right angles on another (inner) tube.

Each of the stands should have an arrow on the leg K by which its position may be read, and the upper face of this leg must be on the same level as the upper face of the scale (Fig. 50).

18. Rule of Signs.

In all formulæ, measurements from the surface of the lens or mirror towards the light will be considered positive, those from the surface away from the light, negative. To apply this, imagine yourself standing at the surface where reflection or refraction is occurring, and turn so that

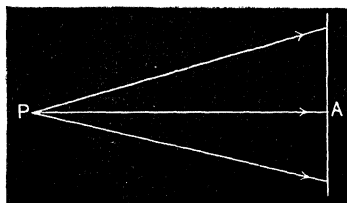


FIG. 54.—Divergent Incident Beam.

the incident light falls upon your face. If by walking forwards from the surface, you come to any point, the distance of that point from the surface is to be considered positive. But if to reach a point it is necessary to turn round and walk in the other direction, that distance is to be reckoned negative.

EXAMPLES.

1. A divergent beam falling on a lens or mirror (Fig. 54).

Suppose A to be the surface of the mirror and P the bright point.

Applying the above rule, we see that the distance AP is positive.

2. A convergent beam falling on a mirror or lens.

If a beam travel in the direction BQ and fall upon the surface A, the point of the cone of light that we are dealing with is Q. To

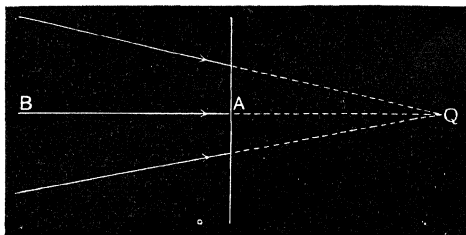


FIG. 55.—A Convergent Incident Beam.

reach this point starting from the surface A, you would have to turn your back upon the light, and thus the distance AQ is negative.

3. Suppose a beam of light falling upon the lens A, to afterwards diverge so that it appears to come from a point R. Then R is the

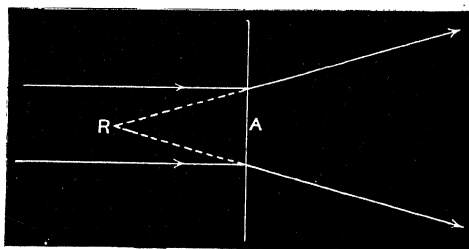


FIG. 56.—Emergent Diverging Beam.

vertex of the cone of rays that is leaving the lens. To reach R from A we should be walking towards the incident light, and the distance AR is positive.

4. Light converging after passing through the lens.

Suppose it to converge to the point S. Then to reach S we shall have to walk away from the light, and the distance AS is negative.

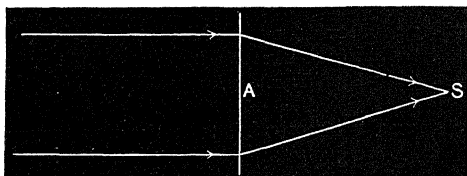


FIG. 57.—Emergent Converging Beam.

In the formula, u always is used to refer to the incident cone of light, and is the distance from the surface to the vertex of that cone. v in the same way is used to refer to the emergent cone—that is, the cone after reflection or refraction as the case may be. So that in Fig. 54, the distance AP is u . In Fig. 55, AQ is numerically equal to u , but being negative, it really is $-u$. In Fig. 56, AR = v , and in Fig. 57, AS = $-v$.

If the incident light is parallel, the emergent light either comes from, or proceeds to, a point which is called the principal focus of the lens, and the distance AR or AS is the focal length. It is obvious from Figs. 56 and 57 that the focal length of a concave lens will be positive and that of a convex lens negative.

Using these conventions, the formula for a mirror is—

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{r},$$

where r is the radius of curvature of the surface; and for a lens it is—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

If we use U, V, F, R, to stand for the reciprocals of these measurements in *metres*, the formulæ become—

$$\begin{aligned} U + V &= F = 2R \text{ for the mirror, and} \\ V - U &= F \text{ for the lens.} \end{aligned}$$

F is called the *power* of the mirror, or lens as the case may be, U and V measure the *curvatures* of the waves at the surface of the lens or mirror, and R that of the surface of the mirror itself, the curvature of a

sphere of radius 1 metre being taken as unity.¹ In words these formulæ express that the sum of the curvatures of the incident and reflected waves is equal to the power of the mirror, and is also twice its curvature; and that the increase of curvature caused by refraction through a lens is equal to the power of the lens. From this way of stating it, it is obvious that if a second lens is placed close against the first one, the curvature will be again increased by the power of that lens, and thus, that the power of two lenses in contact is equal to the sum of the powers of the individual lenses, the power of a convex lens being taken, of course, to be negative. The power of a lens as defined above is said to be so many *dioptries*.

The following way of stating the formula is instructive.

When a spherical wave of light falls normally upon a lens, the change in the curvature is always the same. This change in the curvature is called the power of the lens.

The measurements on the optical bench will be those of u , v , and f , respectively. These must be converted into U , V , F , by the table of reciprocals given at the end of the book.

19. Focal Length of Convex Lens.

i. **First Method.**—*Apparatus.*—Optical bench; cross-wire and receiving screens; flame; 10-inch lens; thin metal rod 6 or 8 inches long, *e.g.* a knitting needle, for the index error; squared paper.

Set up the gas flame, the screen with the hole and cross-wire, also the 10-inch lens and the paper screen in a line, so that the light from the cross-wire after passing through the lens falls upon the paper screen. A bright patch will be seen on the latter. Move this to and fro, and see if an image can be formed upon it of the cross-wire. This will not be possible if the lens is not at a greater distance from the cross-wire than its focal length. To obtain an image on the far side of the lens, the curvature of the emergent light has to be negative. The curvature of the incident light is positive, and if, therefore, the curvature of the incident light is greater than the power of the lens, it would be impossible for that lens to render

¹ Do not confuse *curvature* with *radius of curvature*. If a spherical surface has a small radius, its curvature will be very large; but the surface becomes a plane, and therefore has no curvature, when the radius becomes infinite. The curvature is the reciprocal *in metres* of the radius of curvature.

it negative. The curvature must be reduced by increasing the distance from the lens to the cross-wire until it is less than the power of the lens. By placing the eye close to the lens and looking through it at the cross-wire, some idea may be formed of the behaviour of the light. If the curvature is still positive, it will be possible to see the cross-wire distinctly through the

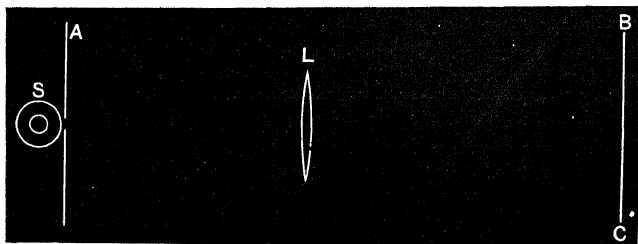


FIG. 58.—Focal Length of a Convex Lens.

lens. It will merely appear magnified. When the curvature becomes negative so that a converging beam is falling upon the eye, it very soon becomes impossible to focus this clearly on the retina, and if the light be received on the screen, the image can be found at a short distance. If the lens is one of very low power indeed, it may not be possible to make the curvature of the incident ray small enough in the length of the bench, and its power will have to be determined by an indirect method. Supposing however the image found, the reading of the scale arrows on each of the stands must be taken.

Index Error.—The difference between the readings on the cross-wire stand and the lens stand would be the distance u , if there were no “index errors.” In the same way, the difference between the readings on the lens stand and the receiving screen stand would be $-v$. But the distance from the screen to the lens will not, as a rule, be exactly the distance from the one arrow to the other: it will differ from it by a constant amount, the “index error.” This constant amount must be found by the steel rod.

Withdraw the lens stand from the cross-wire screen, and insert the rod horizontally between them on a line with the centre of

the lens, and adjust it just to touch the card by the cross-wire at one end and move up the lens lightly to touch it at the other end. Their true distance is now exactly the length of the rod.

Carefully lift out the rod without disturbing anything else, and place it on the scale, with one end exactly opposite the arrow on the stand of the cross-wire screen. If the other end of the rod is exactly opposite the arrow on the lens stand, there is no index error. If not, note how much the distance is from the end of the rod to the arrow. If the needle is longer than the distance between the arrows, the error is positive, and you will write "index A to B 2.3 cms." (say). If the needle is shorter than the distance apart of the arrows, the error is negative. This error is to be added with its proper sign to all the readings of the distances AB, that are subsequently taken.

Repeat the operation with the lens and receiving screen.

A series of readings of the distances u and v can now be taken.

Enter the results thus—

SCALE READINGS.

Cross-wire.	Lens.	Screen.	u	v	U	V	$V - U = F$	f

The column, F, is the power of the lens in *Dioptres*.

U, V, F are found from tables of reciprocals at the end of the book.

The mean of the columns F and f should be taken.

The graphical solution should also be found.

The values of u_1, u_2, u_3, u_4 (Fig. 58) are to be marked off along XO, and the values of v_1, v_2, v_3, v_4 (being negative) are to be measured off along OY. Join the corresponding points. These lines should all meet a line, OF, bisecting the angle XOY, in the same point, F.

The mean position of this point must be found, since the lines will not all meet in OF in exactly the same point. Then the length of the perpendicular FM should agree with the mean value of f found from the last column.

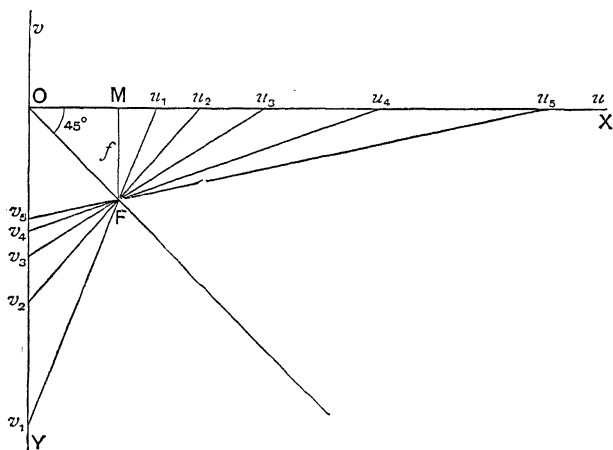


FIG. 59.—Graph of a Convex Lens. FM is the focal length, and σ is negative.

The best results will be obtained when u and v are nearly equal to one another, as the errors of measurement are then less important.

ii. **Second Method.**—*Apparatus.*—As before, with the addition of a plane plate-glass mirror and stand.

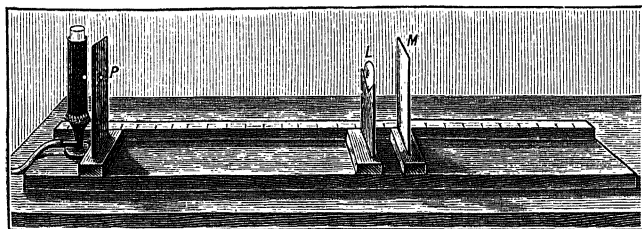


FIG. 60.—Mirror Method for determining the Focal Length of a Convex Lens.

Place a plane mirror, M , close behind the lens, and adjust the distance of the lens L from the cross-wire until the image of the

cross-wire is formed on the screen P itself close by the side of the cross-wire, so that the light is practically returning along its own path. To do this it must be striking the mirror nearly perpendicularly, or as it is a plane mirror, the light falling upon the mirror must be parallel. It has, therefore, no curvature, and the power of the lens must be equal and opposite to the curvature of the incident light. The distance of the lens from the cross-wire must therefore be the focal length of the lens. The distance between the arrows corrected for index error will be the focal length.

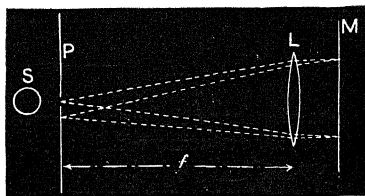


FIG. 61.—Focal Length of a Convex Lens.
Mirror Method.

This is the most accurate method as well as the simplest.

iii. **Third Method.**—*Apparatus.*—The same as for the first method.

If the cross-wire, lens, and screen be adjusted as in the first method, to form an image of the cross-wire P on the screen at Q with the lens at A, it will be found that there is another position,

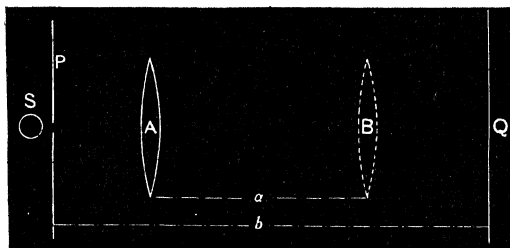


FIG. 62.—Conjugate Positions of a Convex Lens.

B, for the lens at which an image of P is again formed at Q. That is to say, that with the cross-wire P and the screen Q at a

given distance apart, a , there will be two positions of the lens, A and B, at which the image will be sharply formed. Also the distance AP will be equal to the distance BQ, and AQ equal to BP, so that in the formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

the magnitudes of the u and v can be interchanged.

(It must be remembered that v is $-AQ$ or $-BQ$ by the rule of signs, p. 51.)

For this reason the points P and Q are frequently called *conjugate foci* of the lens A.

If

$$PQ = b = AP + AQ = u + (-v),$$

and

$$AB = a = AQ - AP = (-v) - u,$$

$$b^2 - a^2 = -4uv;$$

but

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

or

$$f = \frac{uv}{u - v} = -\frac{b^2 - a^2}{4b}.$$

If, therefore, we measure the distance b between the two screens, and a between the two positions of the lens, we can calculate f .

It will be obvious that the measurement of a will be independent of index errors, as it is merely the distance the arrow attached to the lens stand is moved, and it will not therefore involve any determinations of the exact position of the optical centre of the lens. It does, however, involve index error in the measurement of b . (This method is nearly correct for the measurement of the focal length of an ordinary photographic lens. The following method is theoretically correct for such a lens.)

iv. **Fourth Method: Abbe's Method.**—*Apparatus.*—Optical bench; receiving screen; gas flame; pair of dividers; millimetre scale; in place of the ordinary cross-wire screen, the screen with two holes, vertically one above the other, about

2 mms. in diameter, and 2 cms. apart, with a horizontal cross-wire stretched over each (Fig. 49).

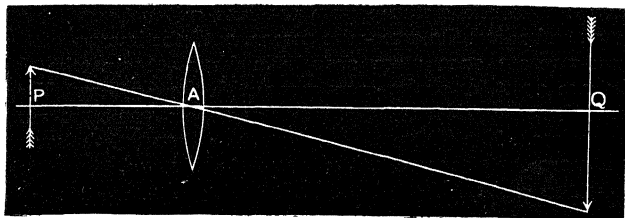


FIG. 63.—Magnification of Image.

When the image of an object placed at P is formed on a screen at Q, the ratio of the size of the image to the object, the *magnification*, is equal to the ratio $\frac{v}{u}$. If this ratio is negative, the image is inverted. This is obvious from similar triangles, if it is remembered that the light passes through the centre of the lens without bending.

Since

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{f-v}{vf};$$

therefore

$$\frac{f-v}{f} = \frac{v}{u} = m_1, \text{ the magnification.}$$

If, without moving the lens, the screen Q be withdrawn a known distance, d , so that v becomes v' , we have—

$$\frac{f-v'}{f} = m_2, \text{ the new magnification.}$$

Subtracting (and remembering that $v = -AQ$, and that m_1 and m_2 are negative quantities) we have—

$$\frac{d}{f} = m_2 - m_1, \text{ or } f = -\frac{d}{m_1 - m_2}.$$

To determine the magnification, focus the two cross-wires on the receiving screen, and set the dividers to the exact distance

apart of the images of the two cross-wires. Measure this distance, a , on the millimetre scale—if possible estimating to fractions of a millimetre. Note the position of the receiving screen; withdraw it a definite distance from the lens *without moving the latter*, and adjust the cross-wire screen, until the images of the cross-wire are once more sharply in focus. Again take the distance apart, b , of the cross-wires with the dividers. Very carefully measure the actual distance apart, c , of the cross-wires themselves. The distance d is, of course, the amount the screen Q is moved as measured by the arrows on its stand.

Numerically, m_1 will be the ratio $\frac{a}{c}$, and m_2 will be $\frac{b}{c}$: thus the focal length is given by—

$$f = -\frac{dc}{b-a}.$$

There are no index errors to be allowed for.

v. **Fifth Method** : For a lens of long focus, that is, a lens of very low power.

Apparatus.—The same as for the first method, and in addition a long focus convex lens and stand (if obtainable, the short focus convex lens is much better replaced by a photographic, or even a lantern objective); squared paper.

Remembering that the power of a lens is the difference between the curvatures of the incident and refracted waves, it will be

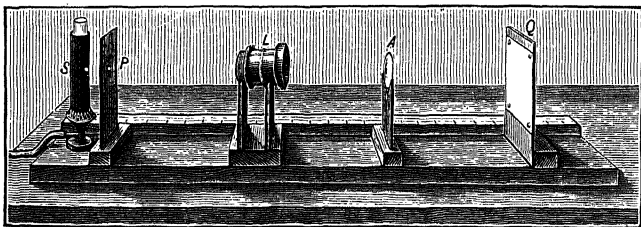


FIG. 64.—Auxiliary Lens Method.

obvious that if a lens be interposed in a converging beam, *i.e.*, one of which the curvature is already negative, it will increase that

negative curvature if the lens is a convex one, and the change in the curvature will be the power of the lens.

Set up therefore the cross-wire screen at P, an auxiliary lens, L (preferably a good photographic objective, or failing that a good achromatic lens), and screen at Q. Adjust the distance so that an image of P is formed at Q. If now a convex lens, A, of low power, be interposed between L and Q, it will increase the curvature of the light falling upon it, and cause it to focus at some point, Q', nearer L.

Note the position of the screen at Q, and then move it forward

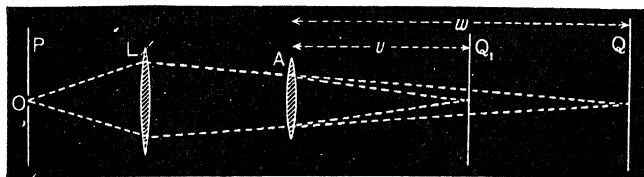


FIG. 65.—Auxiliary Lens Method for a Low-Power Convex Lens.

to Q', again noting its position. Read also the position of A. Find the index error between A and Q.

The curvature of the incident ray was equal to $-\frac{1}{AQ} = U$.

The curvature of the emergent ray $= -\frac{1}{AQ'} = V$.

Therefore the power of the lens, $F = V - U$, can be found.

This method may also be used to find the focal length of any concave lens, and with a good photographic objective as an auxiliary lens, is the best method to employ.

vi. **Sixth Method.**—*Apparatus.*—Telescope ; 10-inch lens and stand ; cross-wire, stand, and optical bench ; knitting needle ; rod for index error.

Focus a telescope on a distant object. Place it to receive the light coming through the lens A from the cross-wire P, and adjust the distance of the lens A from the cross-wire until the cross-wire is seen sharply in focus. If the telescope has been

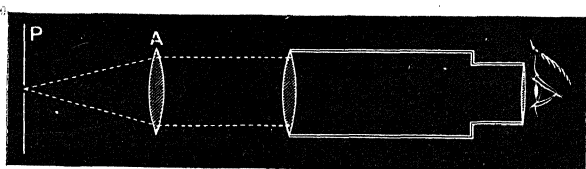


FIG. 66.—Telescope Method of obtaining the Focal Length of a Convex Lens.

properly adjusted, the light entering it will be parallel light, and thus the cross-wire is at the principal focus of the lens A, and a distance AP is its focal length.

20. Focal Length of Concave Lens.

i. **First Method.**—The focal length of a concave lens is best found with the aid of an auxiliary lens, and is similar to Experiment 19 (v.), for a convex lens.

Apparatus.—Optical bench ; cross-wire and receiving screen ; photographic or lantern objective—or, failing this, a convex lens of 6 to 10 inches focus—on a stand ; gas flame ; concave lens and stand ; squared paper.

Set up the screen P and the auxiliary lens L to produce an image of P on the screen at Q. Note the position of this

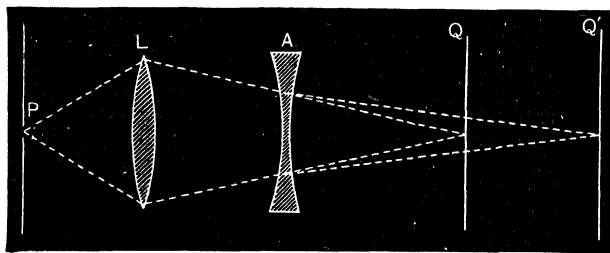


FIG. 67.—Focal Length of a Concave Lens. Auxiliary Lens Method

screen. Withdraw the screen to the end of the bench at Q'. Introduce the concave lens, and adjust it until the image of P is

in focus at Q' . Note the position of the lens A , and find the index error from A to Q . As the light is coming from left to right, the distances AQ and AQ' will both have to be reckoned negative. The vertex of the incident cone of light is on Q . Thus AQ is $-u$ and AQ' will be $-v$.

Several readings must be taken, and tabulated as in Experiment 19 (i.).

The graph will be that shown in Fig. 68.

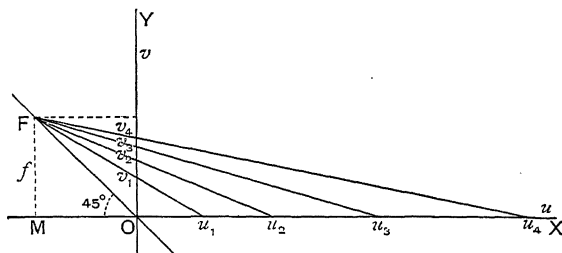


FIG. 68.—Graph of a Concave Lens. FM is the focal length, and is positive.

ii. **Second Method.**—As the power of a lens measures the increase in the curvature of the wave passing through it, two lenses in contact will increase the curvature by an amount equal to the sum of their powers, and this will be, of course, the power of the combined lens. So that if F_1, F_2, F , be the powers of the separate lenses and of the combination respectively, $F = F_1 + F_2$, and if any two of these are known, the third can be calculated.

Apparatus.—Optical bench ; cross-wire and receiving screens ; gas flame ; convex and concave lenses, and a stand which will support them in contact with one another, the convex lens being stronger than the concave ; rod for index error.

Place the concave lens the power of which we wish to find in contact with a convex lens of known power, and determine the power of the combination. We can then calculate that of the concave lens. If the power of the latter is less than that of the convex lens, the combination will act as a convex lens, and we can determine its focal length by any of the methods given for a convex lens, preferably the second method, both as being the

most accurate and the simplest. In the same way, the focal length of the convex lens alone can be found, and thus that of the concave obtained.

As the combination of these lenses will not have negligible thickness, there will necessarily exist some uncertainty as to the

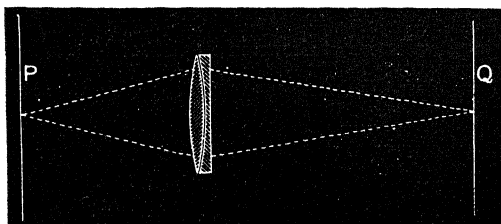


FIG. 69.—Focal Length of a Concave Lens by combining it with a stronger Convex one.

point to which measurements are to be made. If the surfaces of the convex lens are about equally curved, the measurements should be made from its centre approximately, and if the index error is taken from its front surface, half the thickness should be added. If the lenses are of fairly long focal length, and the thickness small, the error produced by neglecting it will not be very great.

In performing the experiment, the two lenses will both have to be mounted in contact on one support so that they may be moved about together.

If there is any appreciable distance between the lenses, the power of the combination is not the sum of the powers of the individual lenses.

The formula then becomes—

$$F = F_1 + F_2 + aF_1F_2,$$

or—

$$\frac{1}{\bar{f}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1f_2},$$

where a is the distance between the lenses, and it is not possible to find either except by the fourth method, p. 59.

When substituting in the formula, it must be remembered that the power of the convex lens is negative. The answer, being the power of a concave lens, should be positive.

iii. Third Method, by the use of a Virtual Image.

Apparatus.—Optical bench ; concave lens ; knitting needle and needle on stands ; plane plate-glass mirror and stand.

Let P be the screen with the cross-wire, or better, a knitting needle in a vertical stand, and A the concave lens. To the right of A, place vertically a silvered mirror, B, covering the

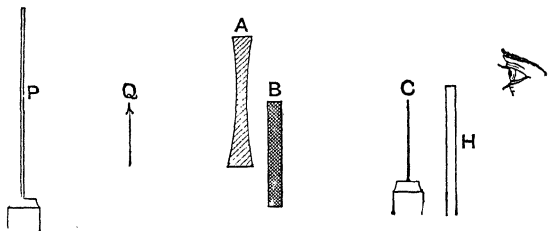


FIG. 70.—Virtual Image Method for a Concave Lens.

lower half of the lens, and facing to the right, so that an eye placed as in the figure will be able to see the reflection in this mirror of a pin at C. This image will, of course, be formed at Q, as far behind the mirror as C is in front of it.

At the same time, through the upper half of the lens, the image of the knitting needle, P, will be visible somewhere between P and A. By altering the position of C, Q, its image, may be formed at any desired point, and therefore may be caused to coincide with the image of P, seen in the upper half of the lens. When these two images coincide, a movement of the eye will not alter their relative position.

To adjust C, therefore, place the eye in such a position that the image of the pin, seen in the mirror, coincides with that of the knitting needle seen in the upper half of the lens. [The image of the pin will be much more easily seen if a piece of white paper, H, is placed behind it so that it appears as a dark line on white surface.] Move the eye to the right or left, and if the images do not remain coincident it shows that they are not really formed in the same position. If on moving the eye to the right the image of the pin moves to the right of the image of the needle, it shows that the former is behind the latter. To improve the result, the pin, C, must be pushed a little closer to

the mirror. In this way it is not difficult to obtain the coincidence of the images of C and P.

The distance AP will be the u of the formula, and AQ will be v . AP can, of course, be read off directly, correction being made for the index error.

$AQ = BQ - AB$, which is $BC - AB$. The readings should be taken from the silvered surface of the mirror, or more accurately still, from the point in the thickness of the mirror one-fourth its thickness from the silvered surface.

iv. Fourth Method, with the aid of two auxiliary Convex Lenses.

Apparatus.—Optical bench; concave lens; cross-wire and receiving screens; the two convex auxiliary lenses, of 6 to 10 inches focal length (half-plate photographic lenses, if possible).

Adjust the first convex lens L so that the light from the cross-wire P shall emerge parallel. This is most conveniently done by placing a plane mirror behind it, as in Method ii. (p. 56). Place the concave lens A to receive this light, and then with the

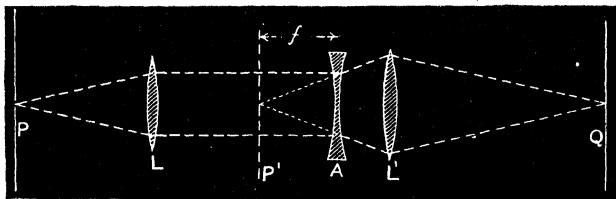


FIG. 71.—Direct Measurement of the Focal Length of a Concave Lens.

second convex lens L' , form an image of P on a screen Q. Note the position of A. Remove A and L and shift P forward to P' until its image is again focussed at Q. Then the light which emerged from A must have been apparently coming from P' , and as the light falling on A was parallel, AP' is the focal length of the lens.

Read the position of P, therefore, and the distance AP' corrected for index error will be the focal length of the lens.

Whether this method will give good results will depend upon the lenses L and L' . They should be telescope- or photographic objectives.

21. Focal Length of a Concave Mirror.

Although the theory of the concave and convex mirrors always precedes that of the lenses, yet as the experimental determinations of the focal lengths of lenses are not only more important but are more easily performed, there is an advantage in placing them first in the practical work. This also allows us to use lenses in determining the focal lengths of mirrors.

i. **First Method.**—*Apparatus.*—Optical bench ; concave mirror ; cross-wire screen.

Adjust the mirror facing the screen P at such a distance that an image of the cross-wire is formed on P , close by the side

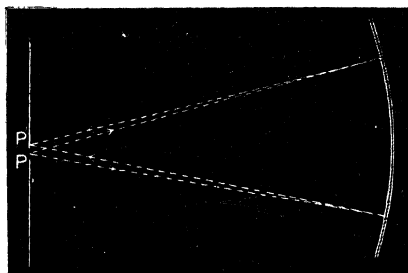


FIG. 72.— Radius of Curvature of a Concave Mirror.

of the wire itself, and that the light which leaves P is returned, after reflection at the mirror, almost along its original path.

If the image and object had perfectly coincided, each ray of light must have been returned exactly upon its path, and thus have struck the surface

of the mirror normally. But lines which are normal to a spherical surface are radii of the sphere of which it is a part. So that the cross-wire is at the centre of curvature of the surface, and the distance from it to the mirror is the radius of curvature of the mirror r , and is double the focal length of the mirror.

This is the most accurate method of determining the focal length of a mirror as well as the simplest. It has the further advantage that the image and object can be kept practically on

the axis of the mirror, that is, on the line joining its centre of curvature to the centre of its surface.

ii. **Second Method.**—*Apparatus.*—Optical bench ; concave mirror ; cross-wire and receiving screens, the latter a disc only about a quarter of an inch in diameter mounted on a wire.

Interpose the small screen between the cross-wire and the mirror (as the inclined mirror is placed in Fig. 75), exactly on the line joining the cross-wire to the centre of the mirror (the axis of the mirror) ; withdraw the mirror until the image of the cross-wire is formed on the screen. The distances from the cross-wire and the small screen to the mirror will be the u and v in the formula for the mirror respectively.

Repeat this experiment with the screen Q at different distances.

The results can be entered thus :—

Position of			PA corrected for index error.	QA corrected for index error.	$\frac{1}{u} = U$	$\frac{1}{v} = V$	$U + V = F.$
P	Q	Mirror.					

The mean of the last column, which should be nearly constant, gives the power of the mirror.

Also obtain the focal length graphically as in Fig. 73.

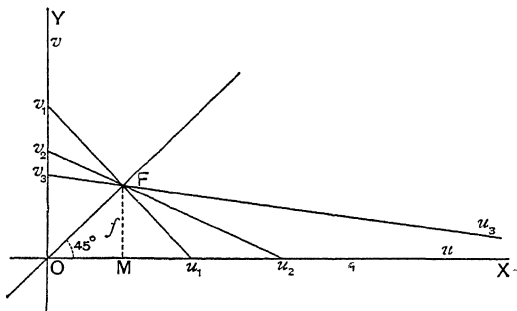


FIG. 73.—Graph of a Concave Mirror. F is the point in which the lines u_1v_1 , etc., meet OF ; FM is the focal length of the mirror.

iii. **Third Method.**—*Apparatus.*—Optical bench; concave mirror; silvered and unsilvered plane mirrors: cross-wire and receiving screens.

Between P and A interpose a flat plate of thin unsilvered patent plate-glass. The light reflected from this glass falls upon the screen Q.

Adjust Q until the image is in focus. The distance of the image will now be the sum of the distances AM and MQ, which can be read with an independent scale.

The disadvantage of this method lies in the fact that there will be a reflection from both the front and back surfaces of the glass plate, resulting in a certain amount of uncertainty in the position of Q. By using a silvered mirror from which some

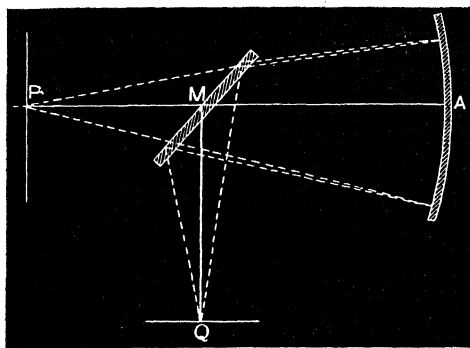


FIG. 74.—Focal Length of a Concave Mirror.

of the silver has been removed over a small area in the centre through which the light from P shall pass, this difficulty will be overcome, since the light reflected from the silvered surface will be much more intense, than that reflected from the unsilvered portion. In this case, the distances AM, MQ, must be measured to the silvered surface, or more accurately, as before, to a point in the mirror a quarter the thickness from the silvered surface.

In the above experiment we have supposed the distance AMQ

to be greater than the distance AP . If the mirror is arranged so that AP is the greater, it will be best to use a small silvered mirror on the axis to reflect the light to the screen Q . In this

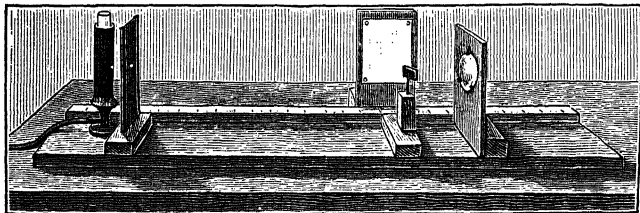


FIG. 75. —Concave Mirror and Small Plane Mirror.

case, the mirror may be of speculum metal, and the distance AMQ can then be found accurately.

This method is introduced as illustrating the principle of the reflecting telescope.

iv. **Fourth Method.**—*Apparatus.*—Concave mirror; optical bench; a small white receiving screen, about 1 inch diameter, on stand.

Allow the light to pass through the lens L , placed at its focal length from P so that a parallel beam falls upon the mirror A .

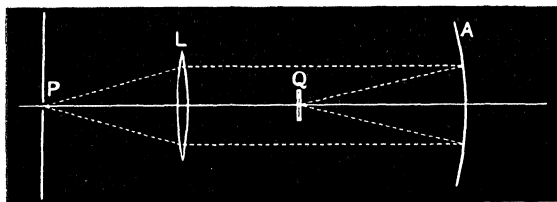


FIG. 76.—Parallel Beam Method for a Concave Mirror.

Receive the reflected light upon a very small screen Q placed on the axis of the mirror.

The distance AQ will be the focal length of the mirror.

If the lens L is a good one (*e.g.*, a photographic objective), this will give good results.

v. **Fifth Method.**—*Apparatus.*—Optical bench ; a needle and a knitting needle, or two knitting needles, on stands.

Place two needles P and Q in stands of the optical bench and put P in front of the mirror. An image of P will be visible in the mirror. If P is close to the mirror this image is erect and magnified.

At a greater distance, it will be seen inverted, and may be either magnified or diminished according to the distance.

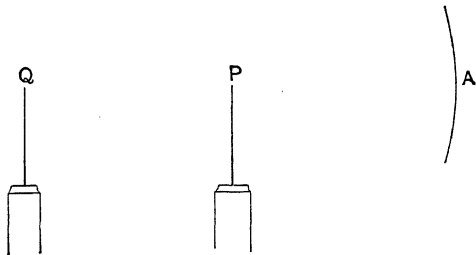


FIG. 77.—Parallax Method for a Concave Mirror.

Place it at such a distance that it is inverted and somewhat magnified. Now put the needle Q in such a position that it appears to coincide with the image of P. That is to say, get P and Q in a line on the axis of the mirror. It may be necessary to rotate the mirror slightly and to adjust the heights of P and Q. The image of P should appear inverted with the point of the needle exactly in a line with, and touching the point of, the needle Q itself.

Move the eye sideways. If the image of P and the needle Q still appear coincident, they are really together. But if they separate, Q must be moved nearer or farther from the mirror until the coincidence is perfect, remembering that the one which moves in the same direction as the eye is moved, is the farther from the eye. The distance PA and QA may now be measured either by using the scale on the optical bench and obtaining the index error, or by holding an independent scale against the surface of the mirror and reading the distances of P and Q directly. They will be the α , u , and v of our formula.

22. Focal Length of Convex Mirror.

The convex mirror does not form a real image of a real object, and thus we cannot obtain its power and curvature directly, but must employ methods similar to those adopted for the concave lens.

i. **First Method.**—*Apparatus.*—Optical bench ; cross-wire and receiving screens ; convex mirror ; a short focus convex lens (preferably a photographic objective).

With an auxiliary lens L, form an image of the cross-wire P on a screen Q. Note the position of Q. Interpose the mirror A between L and Q. The light will now be reflected back and by adjusting A it will be possible to form an image of the cross-wire on the screen P by the side of the cross-wire

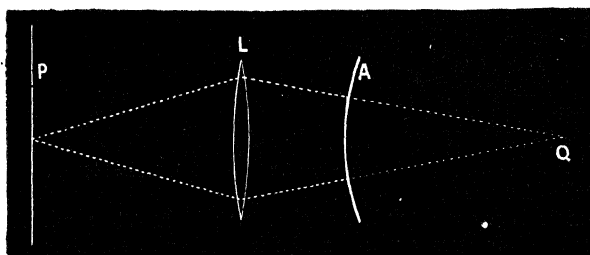


FIG. 78.—Auxiliary Lens Method for a Convex Mirror.

itself. When this is the case, the light being returned upon its path must strike the mirror A normally, and thus the rays must be radii of the surface A and would, if produced, meet at its centre. But they do focus at Q when the mirror is not in the way. Thus Q is the centre of curvature of the mirror, and the distance AQ is its radius of curvature.

If the mirror A has a very small curvature, it will be necessary to put L in such a position that Q is at the end of the bench. Generally the lens should be so adjusted that A is fairly close to it to give the best results.

With a good lens this is the best method.

ii. **Second Method.**—*Apparatus.*—Optical bench ; convex and plane mirrors on stands ; needle and knitting needle on stands.

Place a plane silvered mirror B in front of the lower half of the convex mirror A. At some distance from it place a large needle, a knitting needle for instance, P, in a stand of an optical bench. An eye placed behind this at E will see a

diminished image of the needle formed in the upper half of the concave mirror at Q.

If a small needle P' be placed at a short distance in front of B, an image will be formed by B of this needle, at an equal

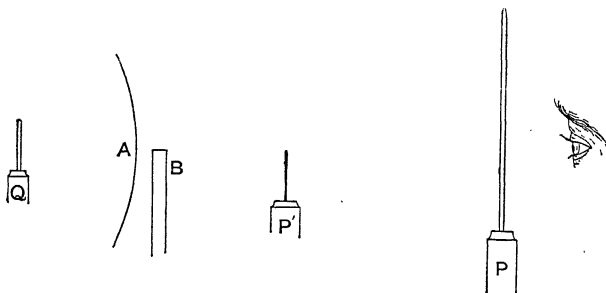


FIG. 79.—Parallax Method for a Convex Mirror.

distance behind it. By adjusting the position of P', it can be so arranged that this image formed in the plane mirror B and the image of P formed in the upper half of the convex mirror A may coincide at Q. This coincidence may be determined as usual by moving the eye to and fro, when no change in the relative positions of the two images should occur.

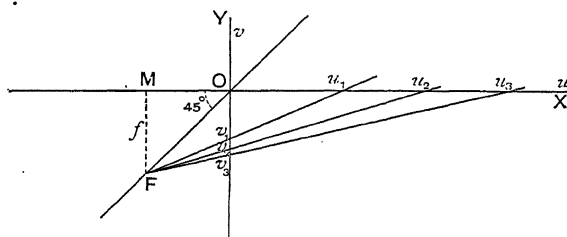


FIG. 80.—Graph of a Convex Mirror. The lines u_1v_1 , u_2v_2 , etc., meet OF in a point. The focal length is represented by FM, and is negative.

Then $AQ = BQ - BA = BP' - BA$.

This is the v of our formula, and is negative.

The distance AP is the u of our formula, and can be measured directly.

Three or four readings should be taken and the results entered in columns as in the case of the concave mirror. (Experiment 21 (ii).) Also obtain the focal length graphically as in Fig. 80.

A piece of white paper between P and the observer will make the images in the mirrors more easily distinguished by providing a plain white background for them.

iii. **Third Method.**—*Apparatus.*—Convex mirror on stand; glass scale and Bunsen clip or stand for same; card on stand; small cork borer to make the holes in the card.

Set the mirror up facing the window.

Let P_1 and P_2 be two of the window bars. A reduced image of these will be formed in the mirror at Q_1 and Q_2 .

Place a glass scale, S, close in front of the mirror, and, standing at a distance of 2 feet away, observe with one eye

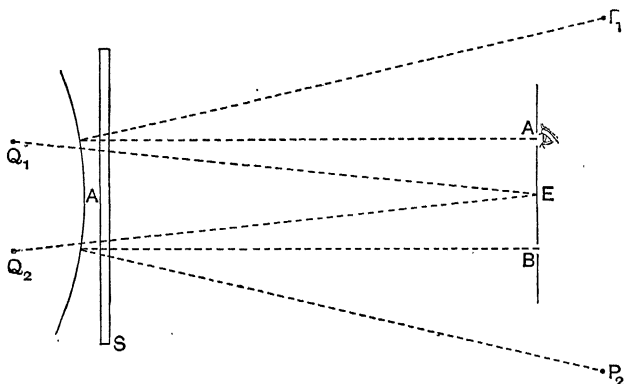


FIG. 81.—Focal Length of Convex Mirror by Virtual Image Method.

the number of divisions subtended by the image on the scale. This will not give the true size of the image owing to the convergence of the rays from Q_1 and Q_2 to E, but it may be treated as a *First Approximation*. Now make two small holes in a piece of card at this distance apart, place the card in a clip

with the holes horizontal, and observe Q_1 through the hole A, and Q_2 through the hole B. The size of the image will now be found to a second approximation, and may be taken as correct. Let it be b . Measure the distance apart of P_1 and P_2 a . Then m , the magnification, which as the image is erect is a small positive quantity, is given by

$$m = \frac{b}{a} = \frac{v}{u} = \frac{f}{u-f}.$$

The distance from A to the window can be easily measured, and therefore

$$f = \frac{mu}{m+1}.$$

Instead of two window bars, a pair of candles may be used.

iv. **Fourth Method for an Unsilvered Mirror**, *e.g.*, one of the surfaces of a lens.

Apparatus.—Convex lens; fusible metal, test-tube, and a beaker of water; rubber ring; metal or cardboard ring.

Place a few sticks of fusible metal in a test-tube and put the latter in a vessel of hot water. In a short time the metal will melt. At the same time warm the mirror carefully by putting it in cold water and gradually adding hot water, so that it may not crack.

Place an indiarubber ring on the surface and press a short piece of brass tube (or a board, in which a circular hole of the required size has been cut) down on this so that it makes a water-tight cell. Pour in the fusible metal. It will very soon set, and, when quite cold, a smart blow with a piece of wood will separate it from the lens. (It is not advisable to try this for the first time with a very valuable lens, as even if the heat does not crack the lens, it is sometimes difficult to separate the two surfaces.)

This metal casting will now be found to make a very good bright concave mirror the curvature of which can be determined by any of the methods given.



FIG. 82.—Method of Casting a Mirror.

v. **Fifth Method.**—To find the curvature of the convex surface of a convex lens.

Apparatus.—Optical bench ; cross-wire screen : convex lens on stand.

Place the lens on the optical bench near the cross-wire.

A patch of light will be seen on the screen P. This is caused by the reflections from the front and back surfaces of the lens. Gradually withdraw the lens. If the surface A, farthest from the screen, is convex, and the lens be gradually withdrawn, the light reflected from this surface will at a certain distance be focussed on the screen P, and, by tilting the lens, the image may be formed close to the cross-wire.

As the light is now reflected back upon its original path, it must be striking the far surface A of the lens normally, and therefore, *in the glass it must be travelling along radii of this surface*. Owing to the refraction at the near surface of the lens, this will not be the case in the space between P and the lens. The ray, as we have seen, is striking the surface A normally, the light which emerges—and the greater part of the light does

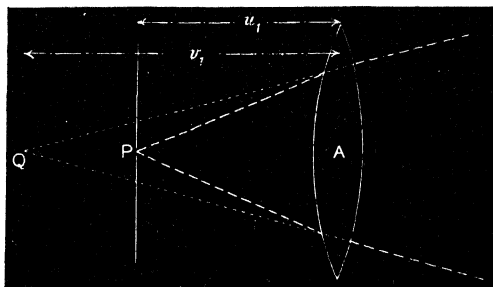


FIG. 83.—Curvature of Convex Surface of a Convex Lens.

emerge—will do so without bending, and the point from which it seems to come must therefore be the centre of curvature of that surface. But the point from which the emergent light appears to come is also the “image.” Thus, when the light which strikes the back surface of the lens is reflected back along its path to P, that light which emerges is coming both from the

image of P and from the centre of curvature of the surface, and these two points are coincident.

Now if we know the focal length of the lens, and the distance of the object, we can, of course, calculate the distance of the image. Therefore :—

(1) Find the distance $AP = u$.

(2) Find the focal length of the lens f (by placing the plane mirror behind it and obtaining the image on the screen as in Method ii. (p. 56).

Then the radius of curvature of the surface AQ, or v , is given by $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ (remember that f , being the focal length of a convex lens, is negative).

vi. **Sixth Method.**—If the curvature of the surface is very slight indeed, it may be found with great accuracy by the method of Newton's rings (p. 157).

23. The Magnification produced by a Lens is equal to the Ratio of the Distances of the Image and the Object from the Lens.

Apparatus.—In place of the ordinary cross-wire screen we shall require one with two holes one vertically above the other about 1 inch apart, each with a single horizontal cross-wire (Fig. 49).

Adjust the short lens so that its centre shall be halfway between the two cross-wires, and form an image of these magnified successively to 1 inch, 2 and 3 inches on the screen. With a millimetre scale measure in each case the exact distance apart of the cross-wires on the screen. Find the distances from the lens to the cross-wires and the screen, correcting for index error as usual. Measure also the actual distance between the cross-wires.

Enter the results thus :—

Position of			u	v	Ratio. $\frac{v}{u}$	Size of		
Crosswire.	Lens.	Screen.				Object.	Image.	Ratio.

24. To find the Refractive Index of Water by the Loss of Power of a Lens placed in it.

Apparatus.—A convex lens about 10 or 12 inches focal length, and a trough with parallel sides large enough to contain it; and the optical bench.

Determine the focal length of the lens in air and then place it in the trough, seeing that the lens is vertical, and against the

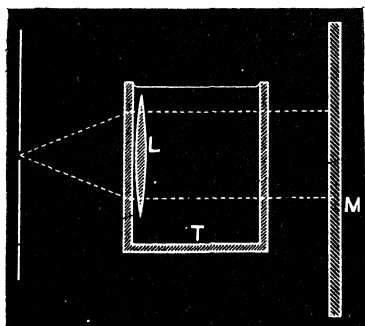


FIG. 84.—Power of a Lens immersed in a Liquid.
L, lens; T, trough; M, mirror.

side of the trough. Fill the trough with the liquid and determine the resultant focal length of the lens as in the figure, using the mirror method. As the light on the mirror side of the lens is parallel, if the lens is placed close to the side of the trough, the thickness of the trough will have no appreciable effect on this determination, and calling f the distance

from the lens to the screen, f is given by the formula—

$$\frac{1}{f} = (\mu - \mu_w) \left(\frac{1}{r} - \frac{1}{s} \right).$$

The focal length in air is given by—

$$\frac{1}{f_1} = (\mu - 1) \left(\frac{1}{r} - \frac{1}{s} \right).$$

So that if the refractive index, μ , of the lens is known, that of the water, μ_w , can be found.

ADDITIONAL EXERCISES ON CHAPTER III

1. Arrange a convex lens and a plane mirror to form an inverted image of the cross-wire on the screen the same size as the original. See that rotating the mirror moves this image; also that the distance of the mirror from the lens is immaterial.

2. Place the plane mirror at a little more than four times the focal length of a convex lens from the screen. Interpose the convex lens. A bright patch will be seen on the screen. Focus this by adjusting the distance of the lens from the screen. It will be found, that when in focus, the image coincides exactly with the original cross-wire, and therefore becomes invisible. Explain the formation of this image.

Breathe upon the mirror, and see that the cross-wire is focussed on the surface of the mirror.

3. See that there is a second position of the lens in which the patch again comes in focus, and that this occurs when the cross-wire is again focussed on the mirror.

4. Move the lens a little so that the image is thrown a little out of focus, and is not quite coincident with the original; by passing a knitting needle over the cross-wire, see that the image of the needle moves in the same direction, and therefore that the image is erect.

5. Slightly rotate the mirror, and see that the image does not move. Explain this with the aid of a diagram.

6. Pair a convex and a concave lens as described on p. 44, Ex. 5. Determine the focal length of the convex lens by the mirror method. (Experiment 19^o(ii).) The focal length of the concave lens will of course be numerically the same. Measure it by one of the methods described in this chapter, and compare your results.

7. Place a convex lens sufficiently far from the screen to form a real image. Place a concave mirror in a line with the centres of the cross-wire and lens, but beyond this image. If the mirror receives the light directly, a bright patch will appear on the screen. Adjust the distance of the mirror until this patch comes into focus. It will be found that when this occurs, the image coincides with the original cross-wire, moving the lens either nearer or farther, causing it to separate on the screen.

9. See if tilting the mirror alters the position. The distance from the mirror to the image is of course the radius of curvature of the mirror.

10. Place a convex lens and a concave mirror in contact. Adjust them to form an image of the cross-wire by the side of the cross-wire. The light striking the concave mirror must now be returning along its path, and therefore it is striking it normally, and must be coming from its centre of curvature. Call the distance from the lens to the screen u . Measure the focal length of the lens, and determine the conjugate focus of the cross-wire by substituting in the formula. The value of v so determined will be the radius of curvature of the concave mirror. It can then be determined directly as in Experiment 21 (i.), and the results compared.

11. Combine a weak concave lens with a concave mirror, and form an image of the cross-wire by the cross-wire itself, as in the last experiment. Measure the distance from the screen to the lens, and the radius of curvature of the concave mirror by Experiment 21 (i). Calculate the focal length of the concave lens by substituting these for u and v respectively in the formula.

12. On the graph of Experiment 19 (i.), set off any distance OA along the line OX, and double that distance OB along the line OY. Join the points A, B, and draw through F a line parallel to AB cutting the axes OX and OY in C and D respectively. Then OC and OD represent the distances from the lens to the cross-wire and receiving screen, at which the image will be twice the size of the object. Adjust the lens and screen to these distances, and test your result.

13. Adjust a concave mirror, the screen with two cross-wires, and the receiving screen to form a real image, as in Fig. 70; and hence measure the magnification produced by a concave mirror. Enter the results as in Experiment 23.

14. Repeat Experiment 22 (ii.), substituting two knitting needles side by side at about 1 inch apart for the single one P, and adjust two needles for the one at P', to coincide with the two images of the knitting needles. Measure the distances apart, both of the needles P, and of the needles P'; use them as the sizes of the object and image, and enter the results as in Experiment 23.

15. Set up a convergent meniscus lens with its convex side facing the cross-wire. As the lens is convergent, it will be possible to obtain an image, by the reflection from the further surface of the lens, upon the cross-wire screen close to the cross-wire, as in Experiment 22 (v.). In this case, as the light is being normally reflected from a convex surface, the light which proceeds through the lens will converge to a real image, and may be received on a screen as usual. The distance from this screen to the lens will be the radius of curvature of the surface. Calculate it from the formula as in the experiment referred to, and compare the results.

CHAPTER IV

OPTICAL INSTRUMENTS

25. The Astronomical Telescope.

Apparatus.—A long focus convex lens 15 to 30 inches focus ; a short focus convex lens of 1 or 2 inches focus, which need only be half an inch in diameter ; stands for same ; knitting needle on stand ; some printed matter at the far end of the room—for instance, a poster.

Set the long focus lens up to face the printing. Stand at some little distance and look into the lens. An inverted image

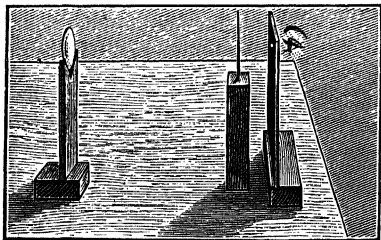


FIG. 85.—Principle of the Astronomical Telescope.

of the printing will be seen. That this image is on the observer's side of the lens can be proved by moving the head, when the image will move across the lens in the opposite direction. Adjust the knitting needle to coincide with this image, by the method of paral-

lax (p. 4). Lastly, place the short focus lens in a line with the other lens and knitting needle, and adjust its distance from the knitting needle so that the latter may be in focus when the eye is placed close against the lens. Then, as the knitting needle has been made to coincide with the image of the distant printing, this printing will be in focus also, and an enlarged and inverted image of a part of the poster will be seen.

26. Galileo's Telescope.

Apparatus.—A long focus convex lens ; knitting needle on stand ; a concave lens half an inch in diameter, and about 2 inches focus.

Adjust the knitting needle to coincide with the image formed by the 15-inch lens as before. Then insert the concave lens between the knitting needle and the long focus lens, at about its focal length from the needle. Remove the needle, place the eye close to the concave lens, and the image will be at once seen, and a slight adjustment of the lens will bring it into focus.

27. The Terrestrial Telescope.

Apparatus.—A long focus convex lens 15 to 30 inches focus ; two short focus convex lenses of 1 or 2 inches focus and half an inch in diameter ; stands for same ; two knitting needles on stands ; some printed matter at the far end of the room.

Adjust the knitting needle as before, to coincide with the image of the poster formed by the long focus lens. Place one of the short focus lenses at about double its focal length from this needle. It will form a real image of the knitting needle on the other side of the lens at about an equal distance. Adjust the second knitting needle to coincide with this image. Lastly, place the second short focus lens at such a distance from the second needle, that the needle may appear sharply in focus when the eye is placed close to the lens. Then, as the needles have in succession been adjusted to coincide with the images of the poster formed by the long focus lens and the first short focus lens, the image of the poster also should be in focus when observed through the second short focus lens. It will be found to be erect. To succeed in these adjustments the lenses must be all exactly in the same line, and care must be taken that the needles are made accurately to coincide with the images.

28. The Compound Microscope.

Apparatus.—Two convex lenses of about 1 or 2 inches focus, and half an inch in diameter ; some small print on a stand ; knitting needle.

Place one of the short focus lenses at about double its focal length from the print. A real inverted image will be formed. (If the image is not inverted, the lens is too close

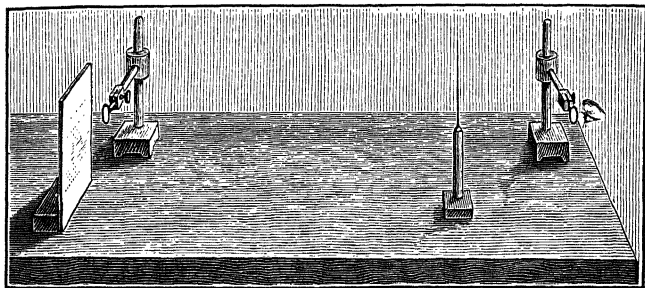


FIG. 86.—Principle of the Compound Microscope.

to the print, and the image will not be real.) Adjust the knitting needle to coincide with this image, and the second lens to view the knitting needle. An enlarged image of the print should be visible at the same time.

The magnification will depend upon the focal length of this second lens and upon the ratio of the distances of the image and object from the first lens. By bringing the print a little nearer to the first lens and withdrawing the second lens to a greater distance at the same time, the magnification can be increased; but with ordinary lenses the image becomes very distorted if the distance of the image is more than two or three times that of the object.

29. Optical Lantern.

Apparatus.—The optical bench with convex lenses of 10" and 6" focal length and about 2" diameter; cross-wire and receiving screen; a lantern slide—preferably one containing black lines and clear spaces, for instance, a diagram; elastic bands; cardboard.

Put the 10" lens, L, at about the middle of the optical bench, and with the receiving screen, S, at the far end find the position at which the cross-wire screen must be placed so that

its image is focussed on the screen S. Clip the lantern slide, B, against the face of the 6" lens, C, with rubber bands, and replace the cross-wire screen by this lens and slide, so that the slide, B, may be in such a position that its image will be focussed on the screen S. Hold a piece of card, or paper, against the surface of the lens L and adjust the cross-wire screen at such a distance behind the lens C that the cross-wire is focussed at L.

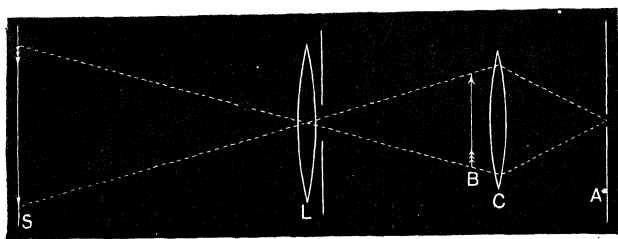


FIG. 87.—Path of Rays through the Optical Lantern.

On removing the paper a clear image of the lantern slide should appear on the screen S, and should be evenly illuminated.

To see the necessity for focussing the cross-wire screen on the lens L, cut a hole about $\frac{1}{4}$ " in diameter in a card and clip the card with a rubber band against the surface of the lens L, so that the hole in the card may be against the centre of the lens.

It will now be found that if the screen A be moved a little forward, the edges of the image on the screen S will become dark; whilst if the screen A be moved back to a greater distance from C, a shadow will form in the centre of the image on S. There is only one position for A in which neither of these shadows occur, and a very small movement either nearer or farther will cause them. As in the ordinary optical lantern the lens C has a much larger diameter, namely about 4 or $4\frac{1}{2}$ inches, this is even more noticeable than when the lens has a diameter of only 2".

If another card be cut having a hole only about $\frac{1}{8}$ " in diameter, it will probably be found impossible to avoid shadows for any position of the screen A. This is due to the fact that the lens C does not converge the light from A again accurately to a

point, so that it cannot be made to go through a hole as small as that mentioned. And with the 4" lens at C, or *Condenser* as it is called, this fault is still more noticeable ; unless the lens L has a large aperture, it is sometimes very difficult to avoid these shadows. When a convex lens cannot be made to converge the light from a point exactly to a point again, the inability is said to be due to *Spherical aberration*.

30. To Find the Magnifying Power of a Telescope.

Apparatus.—Telescope ; metre scale ; a pair of dividers.

i. **First Method**.—Place a metre or 2-metre scale at one end of a room. From the other end observe it through the telescope, and adjust the latter until the divisions on the scale are clearly focussed. Now open both eyes, and whilst looking with one eye through the telescope at these divisions, with the other eye look at the scale itself.

A difficulty will often be found in seeing the scale with the one eye, and the divisions with the other, clearly focussed at the same time. This may be partly due to a difference in illumination, if the objective aperture is too small for its magnifying power. But it is probably due chiefly to the accommodation of the eye.

When observing the divisions in the telescope with the one eye closed, it was possible to see them clearly when the image formed by the telescope was at any distance within the range of vision, that is for a normal-sighted person anywhere between 8 inches and infinity, and the telescope may therefore have been adjusted so that the image of the scale was formed at a short distance. In order to see this clearly, therefore, the eye had to be accommodated to that short distance. When on opening the other eye the attempt was made to see the scale itself at the far end of the room, and its image at a short distance, at the same time, it is obvious that the attempt must fail, as we cannot accommodate one eye to a great distance and the other to a near distance simultaneously. But if, while observing the scale in the telescope, we imagine it to be at the end of the room, and focus the telescope with this idea clearly in our minds (especially if at the same time we open the other eye, and

so fix the accommodation of the eye to that distance), it will be possible to adjust the focus so that the image shall be actually formed at that distance.

The coincidence may be tested by slightly moving the head, still keeping both eyes open. If the image is now really formed at the far end of the room, a motion of the head will not displace it relatively to the objects situated at that distance, and the focus should be adjusted until this is really the case.

If now the image of the divisions of the scale do not quite coincide with the position of the scale as seen by the other eye, the direction of the telescope should be adjusted until they do coincide, and the number of divisions seen in the telescope which correspond to the whole length of the scale must be determined.

The magnification is then obviously the length of the scale divided by the number of divisions it subtends in its image.

ii. **Second Method.**—Focus the telescope on infinity, then pointing it at the sun, receive the emergent light on a piece of paper placed at a short distance from the eye-piece. Place a diaphragm over the objective pierced with two small holes on a diameter a known distance apart d_1 . Two bundles will emerge from the eye-piece which correspond to these holes. Measure on a good transparent glass scale their distance apart d_2 . The magnification m is given by—

$$m = \frac{d_1}{d_2}.$$

Caution.—On no account observe the sun directly through the telescope.

31. Birch's Tangent Lens Gauge.

Apparatus.—The whole instrument can be made from patent plate glass. On to a strip of about 6 inches long and $1\frac{1}{2}$ inches broad at one end are attached, with marine glue, two other strips each about 2 inches long and $\frac{3}{4}$ inch broad, making a small angle with one another. The instrument is shown in Fig. 88. The end view figure (Fig. 89) shows the way in which the glasses are to be tilted to one another. The two inclined plates will touch the lens at two points, and a small system of Newton's rings will be formed at each point of contact.

The apparent distance between these points of contact is to be measured with a reading microscope. This will be the

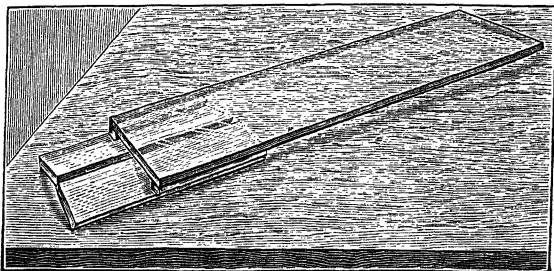


FIG. 88.—Tangent Lens Gauge.

distance a . The true distance between these points will be slightly less than a owing to the refraction through the glass. The amount e to be subtracted will, however, be a constant, which can be measured once for all by measuring the *apparent* distance between two needle points, seen through the glass plates, and their *true* distance apart when the glass is removed. Subtracting this constant amount from a , we obtain the true distance, b , between the two Newton's ring systems. As the angle, θ , (Fig. 90) between the glasses is a constant, if R is the radius of curvature of the surface—

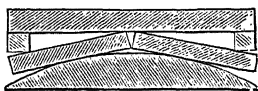


FIG. 89.—Section.

$$b = 2R \cdot \sin \frac{\theta}{2}.$$

and therefore—

$$R = \frac{b}{2 \sin \frac{\theta}{2}}$$

or putting A for $\frac{1}{2 \sin \frac{\theta}{2}}$, which is a constant of the instrument,

we may write $R = A(a - e)$.

The angle θ may be found either by means of a surface of known radius of curvature, or by setting the instrument up

vertically on the spectrometer table, and reflecting the light from the surfaces AB and BC in turn. The angle the telescope of the spectrometer has to be moved through will be 2θ . To be

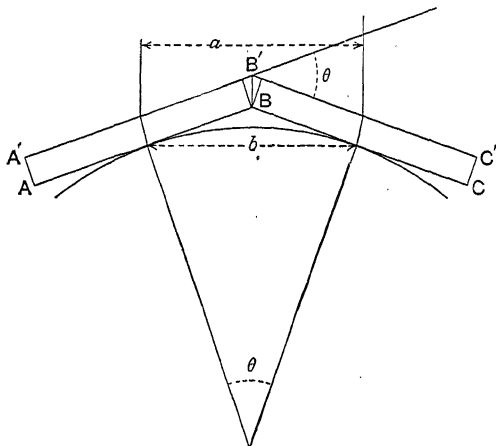


FIG. 90.—Tangent Lens Gauge.

certain that the reflection takes place from the front of the surface AB, and not at the back surface A'B', the latter may be breathed on to destroy the image reflected from it.

If the angle is so chosen that a is fairly large, it may be measured with an ordinary glass scale with sufficient accuracy, and then the instrument becomes a very simple one and will give better results than the ordinary cheap spherometers.

32. The Sextant.

Apparatus.—A sextant, artificial horizon, and metre tape.

The artificial horizon may be a plane mirror or piece of black glass, and if so, it must be mounted on levelling screws, and a level will be needed as well. A mercury bath may be used in place of the mirror and level, if the observations are being made in a place free from vibration; in a room in which other students are working, or in a town, a mercury bath is practically useless.

i. **Adjustments.**—Assuming the circle to be accurately divided, and to be concentric with the arm, there are still several adjustments.

(a) Each mirror must be perpendicular to the plane of the circle.

(b) They should be parallel to one another when the arm is at the zero division.

(c) The axis of the telescope should also be parallel to the plane of the instrument.

To see that the mirrors are perpendicular to the plane of the instrument, look obliquely in the mirror attached to the arm, at the image of the divided circle. If the plane of the mirror is perpendicular to the circle, this image will appear to be a continuation of the circle itself. If not, the image and the circle will be inclined one to the other.

Then, placing the pointer at zero, point the instrument at some very distant object, two images of this will usually be seen in the telescope at the same time, the one transmitted directly through the unsilvered half of the small mirror, and the other formed after two reflections, one from each mirror. If the mirrors are perfectly parallel, these two images will coincide. If they do not, see if they may be made to coincide by a slight movement of the arm, and if this is sufficient to produce coincidence, take the reading and treat it as an index error. If the reading is very much wrong, the adjusting screws of the smaller mirror may be altered. There are usually two of these ; one is the end of a tangent screw which rotates the mirror on an axis perpendicular to the plane of the circle, and this screw will adjust the index error. To adjust it, set the arm at zero by the scale, and turn this screw until the images coincide. If the instrument is held with its plane vertical, a movement of this screw should move the images relatively to one another in a vertical direction. If, while the plane of the instrument is held vertical, the images are separated from one another horizontally, the adjustment must be made by the screw at the back of this mirror, which alters its inclination to the plane of the instrument. The student should never interfere, however, with either of these adjustments ; he should be content with reading the index error.

ii. **Graduation.**—Let M_1 and M_2 be the movable and fixed mirrors respectively, and E the eye-piece of the telescope ; then when M_1 and M_2 are parallel to one another, it is obvious that parallel rays S_1M_1 and S_2M_2 incident on the mirrors M_1 and M_2 , respectively, will, after reflection, coincide in the ray M_2E . If M_1 be turned through any angle, the ray S_1M_1 will be turned through twice that angle ; thus the ray from an object S_3

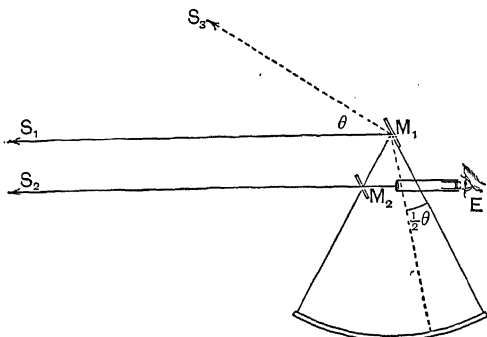


FIG. 91.—Angle between Two Distant Objects.

making an angle θ with the ray from S_1 will be brought into coincidence with the ray from S_1 when the mirror has been turned through an angle $\frac{1}{2}\theta$. In order to read the angle θ directly, the graduations on the arm are always marked in *half* degrees ; thus the angle which is really only 60° is numbered 120° . The readings on the scale give the angle θ directly.

In finding the zero of the instrument, it is necessary that the rays S_1M_1 and S_2M_2 shall really be parallel, and it is useless, therefore, to attempt to adjust the instrument with an object that is not at a very great distance—even a distance of 200 yards is insufficient.

The instrument may, however, be used to find the angle between two objects, S_2 and S_3 , if that angle is fairly large, without appreciable error ; for it is obvious from the figure, that the angle found is the angle subtended at O , the point of intersection of the lines S_3M_1 and S_2M_2 , and therefore, that if the angle θ is of any appreciable size, the position of this point,

O, is practically that of the observer. But when S_3 is very near S_2 , this point may be a considerable distance behind the observer.

iii. Determination of the Angular Elevation of a Point.

Let P be a point and HL the artificial horizon. With a sextant we can measure the angle between the lines PO and OQ. But—

$$\angle POQ = \angle POK + \angle KOQ = \angle POK + \angle PQL.$$

If P be sufficiently distant in comparison with the length of the line OQ, these angles may be supposed equal, and we may assume that the angle POQ is twice the angle POK. If the object is not at a very great distance, the sextant should be used as near the horizon glass as possible, so that the distance OQ may be kept small.

We have first to level the horizon glass. If this is mounted on three levelling screws as usual, the level should first be placed parallel to the line joining two of these screws, A, B (Fig. 93), and the horizon glass adjusted until the bubble is in the centre. The level should then be turned through 180° to see if the bubble is still in the centre. If not, the level is not perfect, and the new position of the bubble should be observed. The correct position of the bubble will probably be half way between this latter one and the former, and the horizon glass should be adjusted until the bubble takes this position. Again turn the level and see whether the bubble now is in the same place. If not, note the mean position, and adjust the horizon glass to get the bubble there. In this way, with a few trials, even if the level is not perfect, the horizon glass can be adjusted so that the bubble is in the same place in its two positions.

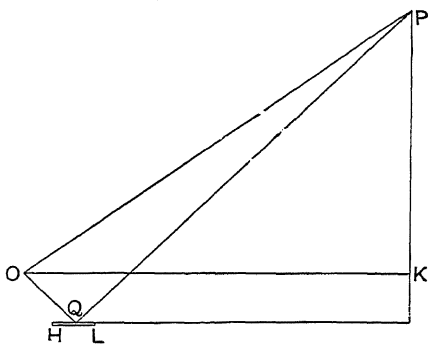


FIG. 92.—Measurement of Altitude.

Now place the level at right angles to its previous position, and adjust the third levelling screw C until the bubble comes to the place which has already been found to be its correct position.

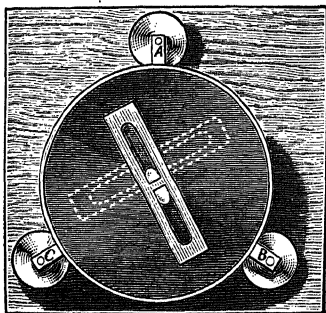


FIG. 93.—Artificial Horizon.

Reverse the level, and see if it remains in the same place. Once more place the level parallel to the line joining the first levelling screws, A, B, and see whether it is still correct; generally a slight adjustment will be necessary, and if so, the third levelling screw, C, must also again be adjusted.

The bubble should now remain in the same place in the level however the latter is turned. If it is intended

to use a silvered mirror instead of a black glass one, as the reflection will take place at the back surface of the mirror, either a piece of mirror whose front and back surfaces are parallel must be selected as described already; or the mirror must be placed in such a position that the series of images formed by it at grazing incidence (p. 33) lie in a direction at right angles to the plane in which the observations with the sextant are to be made, for the want of parallelism will then have the least effect. If now the upper surface be levelled, the lower surface, although not level, will yet be parallel to the horizon in the plane of incidence.

Holding the sextant in the right hand, with its plane vertical, point it down so that the telescope is directed to the image in the horizon glass of the distant point the altitude of which is to be determined, and rotate the movable arm until the image formed after the two reflections in the mirrors of the sextant is brought to coincide with the image formed by the horizon glass. If the horizon glass be only a black mirror, its image will probably be very much fainter than the one formed by the sextant mirrors, and it may be necessary to place one of the dark glasses in the path of the rays forming the latter. If the axis of the telescope can be moved further from the plane of the instrument so as to

receive a greater proportion of the light through the unsilvered half of the small mirror, this will be a better way of equalising the two images. When the coincidence is perfect, the angle can be read upon the scale. To this must be added the zero error of the instrument, if there is one, with its proper sign, and the sum will give the angle POQ. The altitude required is half this.

It is a good exercise to find the altitude of two points vertically above one another, and also to find the angular distance between the two points : the latter should equal the difference between the two altitudes, and thus the observations can be checked against one another.

ADDITIONAL EXERCISES ON CHAPTER IV

1. Place a mirror on a drawing board as in Experiment 1, and insert pins P, Q, R, and S, as in Fig. 1, to appear in a line. Turn the mirror AB through an angle of about 30° , and insert two more pins, R', S', to again appear in a line with the pins P, Q. Measure the angle the mirror has been rotated. Measure also the angle between the line joining the pins R, S, and the line joining the pins R', S'. See that the latter angle is double the angle the mirror is turned through.

2. Make a mirror about 2 inches in diameter with fusible metal, as described in Experiment 22, IV., with a central hole of about $\frac{1}{2}$ inch in diameter, and of from 10 to 20 inches radius of curvature. Make also a concave and a convex mirror of about $\frac{1}{2}$ inch in diameter, and 3 or 4 inches radius of curvature. (These can be cast on ordinary spectacle lenses.)

Set up the larger mirror to form an image of a distant object (a candle flame) on a screen, and so determine the position of the image. Place the concave mirror at a little more than its focal length beyond this image, and adjust it to form an image in the hole in the larger mirror. Examine the latter with an eye-piece.

This forms a model of Gregory's telescope.

3. Repeat the adjustments, replacing the small concave by the small convex mirror, and of course placing the latter mirror at a little less than its focal length in front of the image formed by the larger mirror.

This forms a model of Cassegrain's telescope.

4. Adjust a short focus convex lens (about 6 inches) and a plane mirror to form an image of the cross-wire, as in Experiment 19 (ii.). Rotate the mirror through angles of 5, 10, 15, 20, &c. degrees, and measure

the displacement of the image in each position of the mirror. See if they are proportional to the rotation of the mirror.

5. Replace the combination of convex lens and plane mirror by a concave mirror, and repeat.

Plot the results of both Exercises 4 and 5 on squared paper. Also plot the tangents of half the angle the mirror was turned through (from the table on p. 182); *i.e.* of the angles $2\frac{1}{2}^\circ$, 5° , $7\frac{1}{2}^\circ$, 10° . See that the displacements with the concave mirror are proportional to these tangents.

6. Find the magnifying power of the microscope by comparing the image of a scale seen through the microscope with another scale seen directly at the same time with the other eye, the second scale being placed at a distance of about 10 inches. Of course, in making this comparison, the microscope must be so focussed that the image of the scale seen through it is formed at a distance of 10 inches, and not at infinity; to see if this is so, move the head slightly to and fro sideways—the image should not be displaced relatively to the scale seen with the other eye.

CHAPTER V

DEVIATION AND DISPERSION OF LIGHT

33. Spectroscope.

Apparatus.—Glass prism ; two convex lenses of 10 and 15 inches focus ; stands for same ; metal screen with a slit in it about 1 mm. broad and 1 or 2 cms. long ; screen to receive the spectrum ; gas flame ; and blocks. If the stands for the lenses are not adjustable, the blocks must be of such a size that the centres of the lenses, prism, slit, and the gas flame shall be all one height.

i. **First Method.**—Place the flame behind the slit, and the 10-inch lens at about 12 or 13 inches distant from the slit. Adjust the screen to receive a sharp image of the slit. Interpose the prism. The light will be cut off from the screen. Keep the distance from the prism to the screen the same, and move the screen round until the light which has been bent aside by the prism is once more received upon the screen. It will be found to be coloured. If the distance from the prism to the screen has been kept exactly the same, the upper and lower ends of the coloured band will terminate sharply, the distance should be adjusted until this is the case.

Notice the order of the colours, and see which colour has been most bent aside, red or violet.

Slowly turn the prism round. The coloured band will move sideways on the screen to the left, perhaps, then become stationary, and then move back to the right. See that in the stationary position it is less bent than in any other position. Notice also that in this position the colours are brighter and

clearer than in any other position. This position of the prism is called one of *minimum deviation*. Cut a slit about a quarter of an inch broad in a card, and with the prism at minimum deviation, move the card to and fro across the face of the prism on which the light is falling. Notice that the coloured band, or spectrum as it is called, shifts sideways slightly, showing that the different parts of the prism produce coloured bands on different parts of the screen; thus the band, when the whole prism is used, really consists of several superposed bands, and therefore at any point there are several overlapping colours. The spectrum is

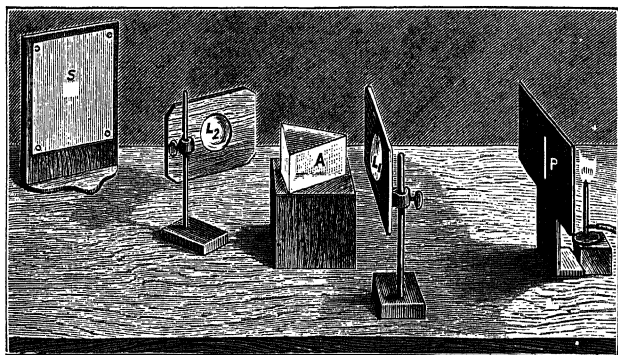


FIG. 94.—Model of Spectroscope.

not *pure*. This is because the light falling upon the prism is not parallel light: it is convergent; and therefore it is falling upon a prism at different angles, and, as we saw by rotating the prism, the deviation produced by the prism depends upon the angle at which the light strikes it. It is therefore impossible with convergent light to obtain a spectrum free from overlapping colours—that is, a pure spectrum.

ii. **Second Method.**—Place the 10-inch lens at its focal length from the slit. To do this experimentally, focus the slit as before on the screen with the lens at about 12 or 14 inches distance from the slit. Gradually withdraw the screen, adjusting the lens at the same time to keep the slit in focus, until the screen has been removed to the far end of the room. In order

to see the image at this distance it will be necessary to shade the screen unless the experiment is conducted in a darkened room. The light from the lens is now very nearly parallel.

Place the prism to receive it. To make sure that the light is really falling on the lens and prism, a piece of white paper may be held in turn against their surfaces.

The prism will deviate the light, and the 15-inch lens is to be put to receive the deviated light. It may be placed at

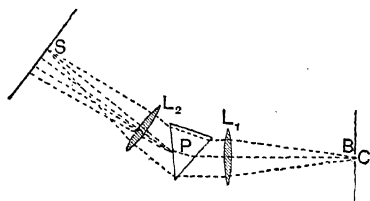


FIG. 95.—Plan of Model Spectroscope. C, slit; L_1 , 10-inch lens; L_2 , 15-inch lens; P, prism; S, receiving screen.

any distance from the prism that is convenient; for as the light from the first lens is supposed to be parallel, the distances of the prism and second lens should not matter; but practically, it is well to keep them fairly close to one another. To see that the light from

the prism is really going through the centre of this lens, again make use of a piece of white paper.

Lastly, adjust the screen to receive the spectrum, and move it nearer to, or farther from, the lens until the colours are as brilliant as possible. On rotating the prism it will be found that the spectrum again moves to and fro sideways, reaching a stationary position at which the deviation is less than at any other—the position of minimum deviation. Set the prism to this position, see that the light is going through the centre of the second lens and that the image is properly in focus.

The colours will be more brilliant than they were in the first method, as the overlapping should now have ceased. To see if this is the case, again slide a card with the slit in it to and fro across the face of the prism. If there is any movement of the spectrum, it shows that the screen has not been properly focussed.

Note that even when the light from L_1 passing through the prism is slightly convergent or divergent, it is still possible to adjust the screen to obtain a pure spectrum (tested by passing a slit across the face of the prism); but it will now be found that the top and bottom edges of the spectrum are out of focus.

34. The Spectrometer.

Apparatus.—The spectrometer consists of a collimator, the function of which is to parallelise the light; the telescope rotating round a graduated circle; and a table also furnished with a graduated circle, upon which the prism or other apparatus is placed.

i. **Essential Parts.**—*The Collimator.* This usually consists of a brass tube varying in length from 8 inches upwards, furnished at one end with an achromatic objective and at the other with a slit. The latter is mounted on a tube sliding in the main tube. The sliding tube should be furnished with a rack-and-pinion motion, by which the slit may be adjusted at the upper face of the lens.¹ The slit must have good edges which are usually made of steel. These should be examined occasionally, and rubbed with an oily rag. A V is usually mounted outside the slit to limit its length vertically. A small right-angled prism is mounted on a movable arm that may be swung in front of the lower half of the slit; and thus the light from a flame placed to one side of the end of the collimator, can be reflected down the tube, at the same time as the light from another flame, directly in front of the tube, passes through the upper half of the slit (Fig. 122). The collimator is usually mounted rigidly upon the stand of the instrument.

The Telescope is an astronomical one, furnished at one end with an achromatic objective of the same diameter as the collimator. The eyepiece required for most purposes is a positive Ramsden eyepiece, which should slide smoothly in its tube and be focussed upon a fine cross-wire. The tube containing the eye-piece and cross-wire is moved by a rack and pinion. The telescope is mounted on an arm which rotates round a vertical axis, and in the best instruments is balanced. The arm in the cheaper instruments is nearly always too weak. The position of the telescope is read on a divided circle by a vernier (or preferably by two verniers at opposite ends of a diameter).

The Table of the instrument rotates, and its position is also

¹ In many instruments a rack is put on the telescope tube and omitted from the collimator. It is useless, however, to be able to focus the telescope unless the collimator can also be adjusted.

read by a vernier on a divided circle. It should have a "hole, slot, and plane" upon which the prism table may be placed. This will allow of the prism table being moved and replaced in the same position.

ii. **Adjustment.**—The collimator and telescope have each to be focussed for parallel light. Point the telescope through a window at a very distant object, and focus it approximately with the rack. Owing to the power of accommodation of the eye, it is possible to see the cross-wire, and the image of the distant object independently, and yet be unable to see them distinctly at the same time. Beginners usually find this the case ; it is because the observer knows that the cross-wire is close to him, and therefore only focusses his eye to view a near object when he wishes to look at it. On the other hand, he will focus his eye at infinity when he wishes to observe the distant object ; with the result that its image will not be formed in the same plane as the cross-wire. He must try to imagine the cross-wire to be stretched over the distant object while he focusses it. In making the final adjustment, it is best to use the method of parallax, and to move the eye to and fro as far as the aperture of the eye-piece will allow, and see if the cross-wire shifts over the image.

If two students are working together at the instrument, they will frequently find a slight difference in their sight, so that after one has focussed the instrument, the other will not find it correct. If, however, the image of the distant object has really been focussed upon the cross-wire by the first observer, it is obvious that the adjustment of the eye-piece, which would bring the cross-wire sharply into focus for the second observer, should make the distant object distinct at the same time ; so that if the cross-wire has once been set at the principal focus of the objective by one observer, *no adjustment of the focussing screw* of the telescope should be made by the second observer. All that he will have to do will be to adjust *the eye-piece* to suit himself, and this will not at all interfere with the adjustment for parallelism. Thus, if each observer in turn adjusts the eye-piece to suit himself, he will be dealing with parallel light so long as the cross-wire (that is so long as the focussing screw) is left untouched.

Now point the telescope directly at the collimator, illuminate its slit, and adjust the length of the collimator tube until the slit

is sharply focussed, again using the method of parallax to make sure that the slit and cross-wire are in the same plane. Then, as the telescope has been focussed for parallel light, that issuing from the collimator must be parallel.

iii. To adjust the Prism so that the Refracting Edge may be Parallel to the Axis of the Instrument.—Let BAC be the prism and A the angle to be determined.

Place it upon the levelling table with one face of the prism, such as AC, perpendicular to the line joining a pair of the levelling screws, S_1, S_2 . Place the table on the spectrometer

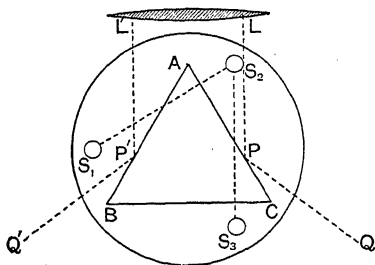


FIG. 96.—Angle of a Prism.

resting in the "hole, slot, and plane," with the edge A opposite the collimator, and the faces AB and AC making as nearly as possible equal angles with the axis of the collimator, so that the light issuing from the collimator may be divided into two approximately equal beams,

one reflected from each face of the prism. Turn the telescope to receive one of these reflected beams. If there is any difficulty in finding the image of the slit formed by this beam, turn the telescope aside and look in the face AC, as in a mirror, when an image of the slit will be visible along the line PQ (this image will generally seem to be indistinct; it is formed by parallel light, and therefore will only be clear to an eye focussed for parallel rays, and as the observer knows the slit is actually at a short distance he finds this difficult to accomplish).

The image of the end of the telescope can always be clearly seen, and from that, the image of the slit may generally be easily found. Then, without moving the head, swing the telescope round until it is in a line with the eye, and the slit should be in its field of view.

If the face of the prism AC is truly perpendicular to the plane of the instrument, the image of the slit will be formed in

the eye-piece at the same height as if viewed directly. If this is not the case, one of the screws must be rotated until it is at the same height. AC, the face which was set perpendicular to the line S_1S_2 , is the face which must be adjusted first.

Now turn the telescope to receive the beam reflected from AB and adjust the screw S_3 , until the image is the right height. It will be seen that the screw S_3 will rotate the face AC in its own plane, and therefore not interfere with its adjustment. It is well to test this by once more turning the telescope to receive the light from AC. If this is not quite correct and a screw has to be moved, the face AB must be adjusted once more with the screw S_3 .

iv. **To measure the Angle of the Prism.—1st Method.**—Turn the telescope to receive the light reflected from the face AB, and read its position with the vernier. Then turn the telescope to receive the light refracted from the face AC, and again read its position. As LP is parallel to $L'P'$, and the angles of incidence and reflection are equal, it is easily seen from Fig. 96 that the angle between PQ and $P'Q'$ is exactly double the angle A; thus the angle of the prism is half the difference between the readings of the vernier when the telescope is at PQ and $P'Q'$ respectively.

2nd Method.—Instead of reading the angle through which the telescope is moved, the telescope may be kept in the position PQ and the prism rotated until the light is reflected from the face AB in the same direction as PQ. This time the angle of the prism will be 180° minus the angle the prism has been turned through. To obtain equally powerful beams for the two reflections, the prism must be drawn back until the edge A is over the centre of the table, as indicated by the dotted lines $A'B'C'$ (Fig. 97).

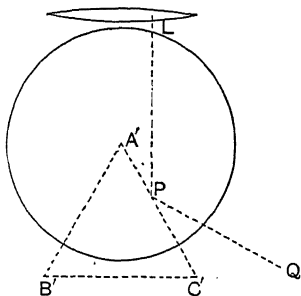


FIG. 97.—Angle of a Prism by rotating the Prism.

As in these experiments the reflections from AC and AB use different bundles of rays from the collimator, it is obvious that

unless the light from the collimator is truly parallel the results obtained will be incorrect ; therefore, not only must the collimator be focussed for parallel light, but its lens must be practically free from spherical aberration. The same thing is true of the telescope.

v. Determination of the Deviation produced by a Prism, and hence of its Refractive Index.—1st Method.—It is shown in books on Geo-

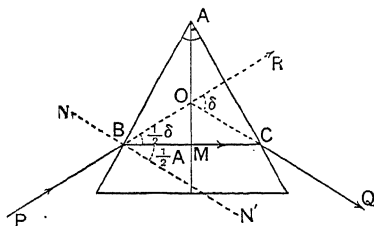


FIG. 98.

metrical Optics that the deviation of a ray of light in passing through a prism is least when it passes through the prism symmetrically.

Let PBCQ be such a ray passing through the prism ABC, and AOM a line bisecting the angle A. Then BC is perpendicular to AM. Produce BO to R ; the angle ROQ is the deviation δ , and, therefore, $OBC = \frac{1}{2}\delta$. If NN' be the normal at B, the angle N'BM is half the angle A of the prism, for it equals the angle MAB.

Now—

$$\begin{aligned}\mu &= \frac{\sin i}{\sin r} \\ &= \frac{\sin \overline{PBN}}{\sin \overline{MBN'}} \\ &= \frac{\sin \overline{OBN'}}{\sin \overline{MBN'}} \\ &= \frac{\sin \frac{1}{2}(\delta + A)}{\sin \frac{1}{2}A}\end{aligned}$$

As we have already determined the angle of the prism, A, if we can find δ , we shall be able to obtain the refractive index from this formula.

To obtain the minimum deviation, δ , produced by the prism, set up the prism on the instrument as directed in Experiment 34 (iii.) with the edge A vertical, and place it so that the light from the collimator falls upon the face AB in some such direction as PB, then the end of the collimator tube will be visible on looking through the prism along the line QO.

Using this as a guide, seek for the slit, the telescope being

turned aside. Notice in what direction it appears to lie (for instance, that it appears to coincide with some object on the wall of the room). Turn the telescope until it is pointing in this direction ; on looking through, the slit ought to be visible. The deviation produced by some of the heavy glass prisms (which have generally a yellowish hue) is very considerable, it is often, therefore, larger than the student expects.

On rotating the prism the image will appear to move across the field. If the rotation is continued and the image followed, it will be found that as the prism is rotated past the position of minimum deviation, the slit moves up to a certain point and then comes back again. This point should be found carefully and the cross-wires placed upon it. The prism will now be in minimum deviation and the position of the telescope must be read upon the divided circle.

Remove the prism, point the telescope directly at the collimator, set the slit upon the cross-wire, and again read its position. The difference between these readings is the deviation produced by the prism.* The refractive index is found by substituting in the above formula.

Another Method, Suitable only for Prisms of Small Angle.—As an exercise it is useful to find the refractive index when one face of the prism is placed perpendicular to the axis of the collimator. As the light from the collimator will strike the prism directly, it may be looked upon as one half of the prism ABC (Fig. 98), namely, such as a prism AMC,—the entrant light being represented by the line BMC, and the emergent ray by CQ. The deviation and the angle of the prism will be half that of the prism ABC. Thus, if δ' and A' be the deviation and angle of this prism, the refractive index is given by—

$$\frac{\sin (\delta' + A')}{\sin A'}.$$

To perform the experiment, point the telescope directly at the collimator and take the readings. Turn it through 90° , using the divided circle, and clamp it. Set the prism up, and reflect the light from the collimator, by the face, AM, down the telescope, and adjust the image of the slit on the cross-wire. Turn the prism through 45° , using the divided circle, so that the face, AM, shall be perpendicular to the collimator. Now

set the telescope to receive the light that has traversed the prism, and so obtain the deviation. The angle of the prism must be found as in the previous case.

vi. To find the Refractive Index of a Liquid.—*Apparatus.*—For this we shall require a hollow prism.

The hollow prism should have the faces movable, so that it can be easily cleaned. It usually consists of a triangular block of glass, through which a cylindrical hole is drilled. On each face

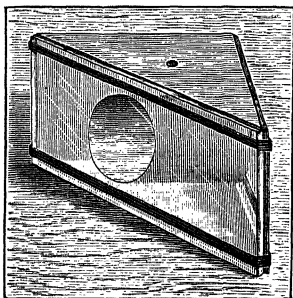


FIG. 99.—Hollow Prism for a Liquid.

a piece of worked glass is held in place by a pair of rubber bands. It is important that these faces shall be of *parallel* worked glass. To see if this is the case, set up the empty prism and look directly through it, and adjust the telescope in a line with the collimator. Remove the prism and see if there is any displacement of the slit. If not, the faces are parallel. If the faces are not parallel, the deviation must be reckoned

from the position of the slit as seen through the prism, and the angle of the prism taken to be that of the central block of glass. If the surfaces of this block are sufficiently well-polished to use them as the reflecting surfaces, it is best to determine the angle as described above, using the reflections from these surfaces. If not, the plates must be put into position, and two images will be seen on each side, one due to the front, and one due to the back surface of the plate. If the prism can be clamped sufficiently securely for the plate to be reversed, the image due to the front surface of the plate will have shifted, and can thus be distinguished from that due to the back surface. Otherwise the prism may be partly filled with water or any other similar liquid, when the image from the back surface will be rendered fainter; or with mercury which will make it brighter, it can thus be distinguished from that formed by the front surface. It is the image of the back surface whose position must be determined in finding the angle of the prism. If the plates are

squares, and the two surfaces, which we are supposing to be not parallel to one another, meet in a line parallel to one edge of the plate, this edge should be made horizontal as the want of parallelism will not then affect the result. It is easy to see in which direction the surfaces will meet together, by observing the images of a distant flame viewed very obliquely, as already described on p. 32. The line of images must be vertical in the prism as put together.

A very fair home-made prism can be constructed by cutting a piece of stout glass tube and grinding with turpentine and coarse emery cloth to an angle of about 60° . Use for the faces two pieces of patent plate selected by the method on p. 34, as nearly parallel as possible. If these plates are not quite parallel, turn them until the line of images they form of a distant flame is perpendicular to the refracting edge of the prism. A small hole for introducing the liquid into the prism should be bored through the top of the tube with an ordinary watchmaker's drill, using plenty of turpentine. The prism must be set up, and its refracting edge adjusted parallel to the axis of the instrument as already described for the ordinary prism.

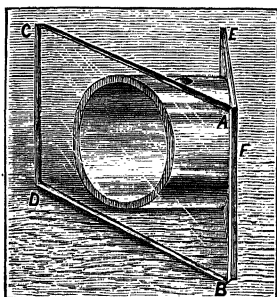


FIG. 100.—Home-made Prism for a Liquid.

Except that the deviation must be measured from the position of the image of the slit with the empty prism inserted, and that the inner faces of the glass plates are to be used in determining the angle of the prism, the measurements are similar to those already described in Experiment 34 (v.).

vii. **To map the Spectrum formed by a Prism.**—By this is understood the determination of the positions in the spectrum of the lines of known wave-length, a curve then being drawn on squared paper, plotting as abscissæ the readings of the instrument, and as ordinates the wave-lengths of the corresponding lines.

The best lines to select will depend upon the source of light

available. The lines of sodium, lithium, thallium, potassium can be obtained with the ordinary Bunsen burner. If an induction coil and hydrogen tube are available, it will give three, in the red, blue, and violet, respectively, which are exceedingly useful. Calcium, barium, and strontium give so many lines that they are practically useless to a beginner, as he is unable to identify them. It is very difficult, indeed almost impossible, for a single student to observe more than the sodium, lithium, thallium, and hydrogen lines. For the potassium ones, two persons are required, one to feed the salt into the flame, and the other to make the observations. The sodium line (or lines if the resolution of the prism is sufficient to separate them) having been determined with the prism at minimum deviation, the next easiest will be the lithium lines. A small quantity of lithium chloride is to be placed in the flame, either by heating a piece of platinum wire, and dipping it whilst still hot into the salt, when sufficient will adhere to the wire for the purpose, or the salt may be placed upon a piece of gauze in the same way as the sodium is usually supplied. A red and an orange line are usually visible. The Bunsen flame must be supplied with a large amount of air so that it may give a high temperature, or the salt will not be volatilised. Potassium gives two lines, a red and a violet one ; neither of these can be seen at all easily. The best results can be obtained with nitre. A piece of fine straight wire is heated and dipped into powdered nitre, then held in the edge of the flame. The nitre will melt to a kind of bead, and then suddenly burst into flame. It is only while this flame lasts that the lines are bright enough to be seen. The violet line is so far in the extreme violet that students often do not move the telescope far enough to get it in the field. It is only after a large number of these flares has been produced that the readings can generally be obtained. The lines are very valuable to a beginner, as being quite unmistakable and giving the positions of the extreme ends of the spectrum. For the other metals mentioned, the heated wire is dipped into its chloride and held in the flame. Thallium gives a very useful line in the green, and strontium in the blue.

If an arc lamp is available, it should be focussed upon the slit with a lens (its own condenser will do). The carbons must be set up vertically and separated considerably so that

an image is formed of the space between the carbons. The salts may then be fed with a spare carbon. Lithium in the arc gives a line practically identical with the blue strontium line. Magnesium gives three nearly identical lines in the green. Neither of these is visible with the ordinary Bunsen, as the temperature is not high enough.

For the hydrogen lines, the hydrogen tube must be connected to an induction coil, and mounted vertically so that its narrow part may be opposite the slit. On working the coil, the hydrogen lines will be very easily found, and their positions can be read, then the curve may be drawn through the lines obtained.

Enter the results thus :—

Line.	Reading.	Wave-length. λ cms.	Square of Wave Number. $\frac{1}{\lambda^2}$	Source of Light.
Na, D_1 . . .	—	$5890 \cdot 10^{-8}$	$2883 \cdot 10^5$	Bunsen
Na, D_2 . . .	—	5896 "	2876 "	"
Li, red . . .	—	6708 "	2222 "	"
orange. . .	—	6104 "	2684 "	Arc
blue . . .	—	4602 "	4722 "	"
K, α red . . .	—	7665 "	1702 "	Bunsen
γ violet . . .	—	4044 "	6115 "	"
H, α red . . .	—	6563 "	2321 "	Vacuum tube
β green . . .	—	4861 "	4232 "	" "
γ blue . . .	—	4341 "	5307 "	" "
Tl, green . . .	—	5350 "	3492 "	Bunsen
Mg, green . . .	—	5184 "	3721 "	Arc
" . . .	—	5173 "	3737 "	"
" . . .	—	5168 "	3744 "	"
Sr, blue . . .	—	4607 "	4710 "	Bunsen
Ca, blue . . .	—	4227 "	5597 "	"

On a piece of squared paper plot a curve with the readings of the instrument along the horizontal line and the wave-length vertically ; also plot a curve on a second piece of squared paper giving the readings horizontally, and the squares of the wave numbers vertically. This curve will be found almost a straight line, and therefore is much more easily drawn accurately. Any reading that is not very approximately on the line should be repeated, as it indicates an experimental error. With the aid of either of these curves, preferably the latter, the wave-length of

any unknown line can be determined, and therefore the line can be identified: if, for instance, strontium or barium be put in the flame, each of which gives a large number of lines, the readings for some of these lines may be taken and the lines identified by reference to a table of wave-lengths.

ADDITIONAL EXERCISES ON CHAPTER V

1. Stick a narrow strip of white paper on a black card. Hold a prism close to the eye, parallel to this strip, and look at the strip through it. Make a drawing of the appearance, indicating which edge of the strip appears red, and which blue. Draw a diagram of the course of rays through the prism to explain the colour.

2. Draw a thick black line on a white card, and examine it with a prism held parallel to the line. Make a drawing showing which edge of the line appears red, and which blue. Draw a diagram to explain this appearance.

3. Look at a distant window through a prism, and slowly rotate the latter. The window sometimes appears coloured and more or less indistinct: at other times, perfectly distinct, and free from colour. In both cases, the image moves as the prism is rotated. Note, and explain, the direction of motion in each case. Has it anything to do with minimum deviation?

4. Replace the screen in Experiment 33 (ii.) by an eye-piece, or a very short focus lens. The spectrum, as seen through either of these, will be found exceedingly brilliant.

5. Replace the screen by a plane mirror, or, better, by a concave mirror, of a radius of curvature equal to the focal length of the second lens. The light forming the spectrum will be reflected back. Turn the mirror until the reflected light passes once more through the system. It will be focussed on the screen; the image will practically coincide with the original slit, and should be free from colour. Rotate the mirror—it will not displace this image.

Between the slit and the first lens interpose a silvered mirror at an angle of 45° to receive about half the beam. This part of the beam will now be reflected off at right angles, and may either be received on a screen, or examined in an eye-piece. It will be practically white, although, owing to the imperfection of the lenses, the edges will be coloured.

Pass a knitting needle slowly across the spectrum near the mirror. It will cut off the colour in a part of the spectrum only; the image of the

slit will become beautifully tinted, being formed by those colours which are left.

6. Replace the screen of Experiment 33 (ii.) by a convex lens, of large enough diameter to include the whole of the spectrum; place the screen at a little more than its focal length distant from this lens, and adjust it at the conjugate focus of the prism. In this position, a rectangular patch of light will be formed upon the screen, which should be white, and evenly illuminated. (If only the *vertical* edges are coloured, the colour can generally be removed by slightly inclining the lens.)

Pass a knitting needle over the surface of the lens, the colour of the patch will change as the knitting needle obstructs different parts of the spectrum. Should sufficient light not be obtained to see the patch on the screen, an eye-piece may be substituted.

7. Place a piece of coloured gelatine in the path of the light through the spectrometer. A dark band will be formed in the spectrum, whose position and extent will depend upon the colour of the gelatine. Set the cross-wire of the telescope, successively, upon the places where this band begins, and ends, and take the scale readings. Estimate as nearly as possible the diminution of brightness caused by the gelatine along the spectrum. (This is the most easily done if the gelatine is placed close in front of the slit and covering only its lower half.) Plot the scale readings horizontally, and let the vertical height represent the apparent brightness of the spectrum.

CHAPTER VI

ILLUSTRATIONS OF METHODS OF DETERMINING THE VELOCITY OF LIGHT

Velocity of Light.—It is impossible, of course, to determine the velocity of light without very special apparatus : but the principle of the various methods can be much better understood if the optical parts are adjusted in each case, and this can be done without any special apparatus. Especially in the case of Foucault's method, the ordinary student usually finds that the adjustments of the apparatus are by no means so easy as they appear from cursory reading in the text-book, and that actual experimenting with the rotating mirror greatly assists a proper comprehension of the theory.

35. Fizeau's Method for Determining the Velocity of Light.

Apparatus.—The source of light may be the screen with the small hole and the burner used with the optical bench ; a lens of 6 or 8 inches focus ; a piece of very thin plane parallel glass about half an inch broad (or a flat micro cover glass selected as described on p. 34) ; a pair of lenses to form the collimator and telescope of about 10 or 12 inches focus ; a positive low-power eye-piece, or failing that, an ordinary small convex lens of about 2 inches focus ; a plane mirror ; a cardboard disc with a large number of teeth equal in breadth to the spaces between them—mounted on an axis that may be easily rotated, or better, it may be turned by a small electro-motor.

Adjustments.—Measure the focal lengths of the lenses that are to be used for the collimator and telescope respectively. Mount them some distance apart on a table facing one another.

Place the rotating disc exactly at one of the focal points of the lens L_1 (Fig. 102). A little behind it place the unsilvered mirror at an angle of 45° . With the 6-inch lens form an image of the hole in the screen A, illuminated from behind, upon a tooth of the disc at a . Turn the sector until a space allows the light to pass through to the lens L_1 . It will emerge from L_1 in a

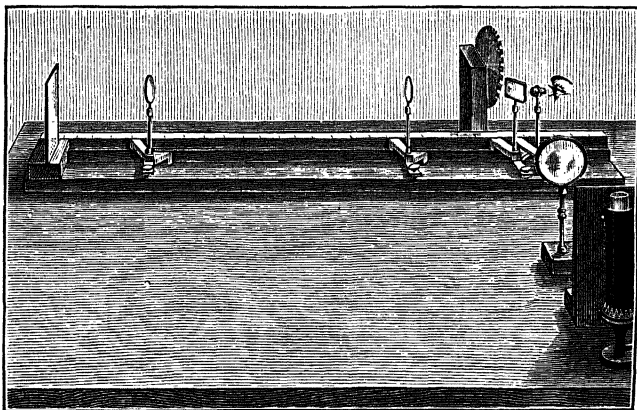


FIG. 101.—Arrangement of Apparatus to illustrate Fizeau's Method of determining the Velocity of Light.

parallel beam, since A was at the principal focus of L_1 , and so reach L_2 . The lens L_2 will form an image of the source of light at its principal focus. Hold a piece of paper here to find its position, and then place the mirror M in the position occupied by the paper, so that the light may be reflected back upon its course. If the mirror is normal to the common axis of the lenses, the light will again pass through the lenses L_2 and L_1 , and be converged to A. From here a part of it will be reflected by the parallel mirror back to the source, but part will go through and may be received in an eye-piece, E. (The eye-piece must be of sufficiently low power to allow room between it and the disc for the unsilvered reflector.)

On rotating the disc, the light will alternately be cut off or allowed to pass as the teeth and spaces pass A.

When the speed is increased, persistence of vision will cause the light to appear uniform. If the speed of the disc could be sufficiently increased for a cog to have time to occupy the position of a space while the light travels from A to the mirror M and back, the light which

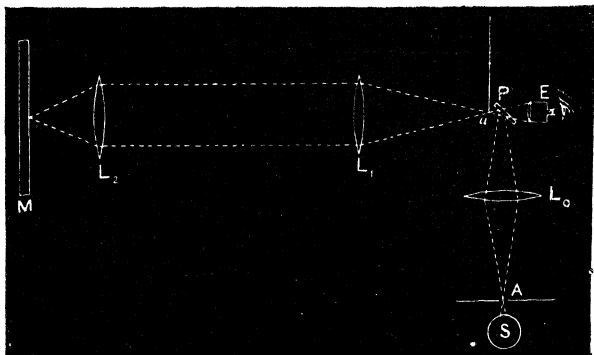


FIG. 102.—Path of Rays through the Apparatus illustrating Fizeau's Method of determining the Velocity of Light.

passes through each space would be cut off by the succeeding cog, and darkness would result. It is, of course, quite impossible to reach this speed. In the actual experiment, the distance from A to M was rather over 17,000 metres, and then with 720 teeth on the disc, the speed was able to be made sufficiently high.

The time from A to M will be $\frac{d}{V}$, where V is the velocity of light, and d this distance, so that in the time $\frac{2d}{V}$, the tooth must move into the position occupied by a space. If there be m teeth on the wheel, this means $\frac{1}{2m}$ of a revolution, and if the wheel rotates n times per second, it will occupy $\frac{1}{2mn}$ of a second for this movement.

Therefore, when darkness occurs—

$$\frac{1}{2mn} = \frac{2d}{V}, \text{ or } V = 4mn \cdot d.$$

If the speed of the disc be still further increased, the light will

appear again, and will reach its maximum brightness when a space has time to take the place of a space while the light goes from A to M, and back, and V is given by $2mn'd$, n' being the number of revolutions a second.

Now as the speed is increased, the brightness dies away and grows again gradually. If teeth and spaces are all equal in width, the diagram of brightness is made of straight lines as in Fig. 103.

Fizeau, in 1849, with this apparatus found the velocity 315,364 kilometres a second in air.

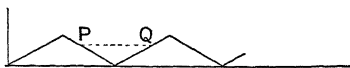


FIG. 103.—Curve of brightness.

Cornu, by employing a small wheel of only 2 or 3 cms. diameter, was able to give it a much higher speed. He also placed the telescope at a greater distance, so that he was able to obtain extinctions of a higher order, so that the light which went through a space returned upon the successive teeth up to the 21st. His collimator and telescope were also of very large diameter, 15 cms. for the telescope and 37 cms. for the collimator lens.

Messrs. Young and Forbes used two telescopes at distances in the ratio of 12 and 13, and observed when the light that was fading in the one was equally bright with that growing in the other, as at P and Q on the curve of brightness (Fig. 103), they were able to determine with more accuracy the speed at which this occurred, than the speed at which the light was just extinguished.

36. Foucault's Method for Determination of Velocity of Light.

Apparatus.—For the rotating mirror, the mirror supplied for a manometric flame apparatus will do very well. Failing this, a plane mirror about 4 inches square can be mounted roughly on a vertical axle. A source of light ; a screen with a hole in it ; an unsilvered reflector ; an eye-piece ; a convex lens of about 10 or 12 inches focal length ; and also a concave mirror of long focal length—2 or 3 feet if possible.

Adjustments.—Measure the focal length of the mirror. Set up the screen A and adjust the lens to form an image of A at a distance of 5 or 6 feet on a screen, S. At a distance from S equal to the radius of curvature of the concave mirror, place the centre of the rotating mirror M. Place the concave mirror

as in Fig. 104, so that its centre of curvature coincides with the axis of rotation of the revolving mirror. Adjust the lens to focus the light from A upon the surface of the concave mirror, after reflection in the rotating mirror when this mirror is turned into the right direction. Then, it ought to return upon its own path, after reflection in the concave mirror, striking the rotating mirror again at the same angle, and forming an image of A coincident with itself.

Place the unsilvered reflector in the path of the rays between A and L, and some of this light will now form an image at B. Adjust the eye-piece behind B to view this image. Rocking the rotating mirror ought now to produce no displacement of the image B; and if the mirror is rotated, an image of A ought

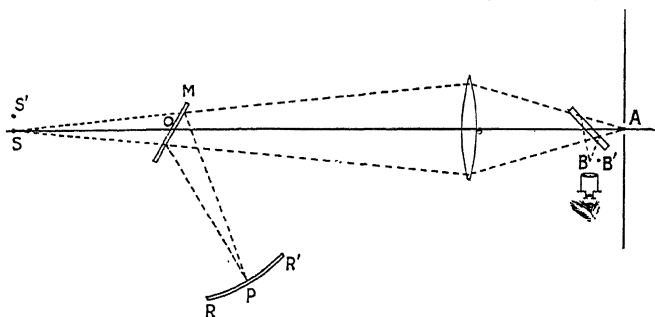


FIG. 104.—Path of Rays through Apparatus illustrating Foucault's Method of determining the Velocity of Light.

to be visible once in each revolution, during the time that the light falls upon the concave mirror RR' ; if the mirror be rotated sufficiently rapidly, a faint image should be visible at B the whole time.

In whatever position M may be, if the light falls anywhere upon RR' , the image formed on RR' will always be the image in the mirror M of that at S, so that the apparent position of the image in R is always at S. But if the speed of rotation of M could be made sufficient, the light which forms the image P in R which is reflected from the mirror while in one position, would, after travelling from M to P and back, find the mirror had moved, and therefore the image of P in the mirror would appear at a point, S' , a little to one side of S, and the conjugate

image B' would have moved to B'. As the light will be deflected through twice the angle θ , the angle SOS_1 will be 2θ .

Let the distance MP be d , then—

$$SS_1 = 2\theta d. \quad \dots \dots \dots (i.)$$

If t be the time taken by the light to pass from M to P and back—

$$t = \frac{2d}{V}. \quad \dots \dots \dots (ii.)$$

If n is the number of revolutions per second of the rotating mirror at this moment, the angle described in one second will be $2\pi n$, and therefore, in the time t , it will be given by—

$$\theta = 2\pi nt. \quad \dots \dots \dots (iii.)$$

Let $AL = a$, $LM = b$; then as BB' and SS' are conjugate with respect to the lens L—

$$\begin{aligned} BB' &= SS_1 \left\{ \frac{a}{b+d} \right\} \\ &= 2d\theta \cdot \frac{a}{b+d} \text{ from (i.)} \\ &= 2d \cdot 2\pi nt \cdot \frac{a}{b+d} \text{ from (iii.)} \\ &= \frac{8\pi nadt^2}{V \cdot (b+d)}, \text{ from (ii.),} \end{aligned}$$

from which V can be found.

Foucault made the distance d about 20 metres, and used a turbine to rotate his mirror, the latter being very accurately balanced on its axis of rotation that it might rotate without shake. He was unable to place the concave mirror at any great distance from the rotating one, owing to the loss of light, for the light will only be reflected while the mirror M moves through an angle equal to $\frac{1}{2}ROR'$, and thus, as RR' is placed at a greater distance, the light becomes fainter and fainter.

37. Michelson's Method for Determination of Velocity of Light.

Michelson, in 1879, used a slight modification of Foucault's method, replacing the concave mirror by a plane mirror and placing a convex lens between the two mirrors.

Apparatus.—To imitate this experiment we shall require to replace the concave mirror by a plane mirror and a convex lens of as long a focal length as possible.

Measure the focal length of the lens, and place it so that it may be this distance from the axis of rotation of the rotating mirror M , so that light which diverged from this axis would emerge from the lens as a parallel beam, and would fall normally upon the plane mirror if this is placed perpendicular to the axis of the lens. Place the screen A at a short distance from the revolving mirror. Then, as the distance AOC is greater than

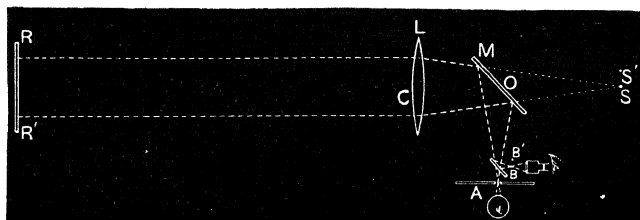


FIG. 105.—Path of Rays through Apparatus illustrating Michelson's Method of determining the Velocity of Light.

the focal length of the lens, a real image of A will be formed on the other side of the lens. The reflecting surface of the mirror RR' must be made to coincide with this image.¹ To do this put a screen to receive the light emerging from L , and adjust it until the image of A is clearly focussed. Then replace the screen by the mirror RR' . See that the light from RR' passes again through the lens L . Insert a plane unsilvered mirror to reflect the light to B , and examine the image with an eye-piece as before.

If $AO = a$, $OC = b$, and $RL = d$.

The time taken from M to R and back will be—

$$\frac{2(b + d)}{V} = t.$$

During this time the mirror will move through an angle θ , given by—

$$\theta = 2\pi nt,$$

n being the number of revolutions per second; and therefore AA_1 is the displacement of the image—

$$AA_1 = 2\theta \cdot a = 4\pi nt \cdot a = \frac{8\pi na(b + a)}{V}$$

As this image would be visible during the whole time that the light from M falls upon the lens L, the distance RR' , which depends merely upon the distance from A to M, can be increased indefinitely without appreciable loss of light.

Michelson made this distance about 2000 feet, and using an air turbine to drive M, he was able to obtain a displacement of the image AA' of more than 13 mms.

ADDITIONAL EXERCISES ON CHAPTER VI

1. Measure the distances apart of the pieces of apparatus of Experiment 35. Count the number of teeth, m , on the wheel: assume the velocity of the light given in the text, and calculate the number of revolutions of the disc that would be required to exactly extinguish the light.

2. Measure the distances a , b , d , in Experiment 36, and calculate the number of revolutions of the mirror per second that would be required to produce a displacement, BB' , of 1 mm. in the position of the image.

3. Measure the distances a , b , d , in Experiment 37, and calculate the number of revolutions of the mirror per second that would be required to produce a displacement, AA' , of 1 mm. in the position of the image.

CHAPTER VII

PHOTOMETRY

THE basis of the photometric comparison of two lights is the law that the intensity varies inversely as the square of the distance. This is true only if the screen upon which the light is to be received is illuminated solely by the direct lights and receives no scattered or reflected light from surrounding objects in the room itself. It is obvious, for instance, that the light transmitted along a perfectly reflecting tube would remain constant at all distances. It is therefore necessary in photometric work to have a room in which the reflections can be eliminated. The walls of the room should be painted a dull black, and the table covered with a black cloth. The instrument should also be blackened wherever reflections are likely to occur. Some photometers are much more liable to error in this respect than others. The grease spot photometer is perhaps most sensitive to scattered light. A photometer such as Rumford's, in which the ordinary scattered light from a room will equally affect the two shadows that are to be compared, will only give false readings to the extent in which either of the lights is reflected upon the screen,—for instance, if it has a polished base.

38. The Grease-Spot Photometer.

Apparatus.—Materials for constructing a grease-spot photometer as enumerated below.

i. Let AB be a piece of white paper, and CD be the grease spot. Then S and S_1 are two light sources. It is usually stated that the light from S which falls upon AB will be chiefly transmitted through the semi-transparent grease spot CD, and will be scattered from the remainder of the paper; so with the

light S_1 which falls upon the other side. If, therefore, the distances of S and S_1 are so chosen that the light on each side is equally intense, that part of the light of S which

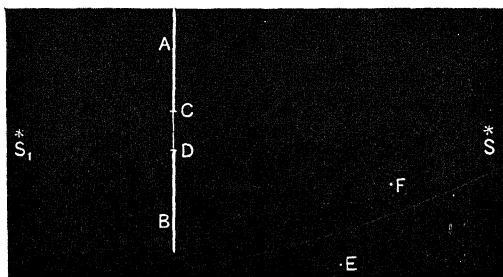


FIG. 106.—Grease-Spot Photometer.

passes through CD , and is, therefore, lost as scattered light to an eye placed at E , will be made up by the light which goes through the same spot CD from S_1 . No doubt the amount of light leaving the area CD from the two sources together must be the same as that leaving the remainder of the screen, but with an ordinary grease spot, the amount which reaches the eye placed at E will not be the same. The light which passes through the grease spot is not equally scattered in all directions, but is much more intense along the axis, and falls off rapidly away from the axis, whereas the light scattered from the remainder of the paper is scattered far more equally in all directions, so that the brightness of the spot varies very considerably according to the position of the eye. If, for instance, S_1 and S_2 are equally strong lights, and the grease be placed half-way between them, the spot will appear comparatively dark, when the eye is at E , whilst at F it would appear very much brighter than the remainder of the paper. Any grease spot is, therefore, untrustworthy and practically useless. The fault, of course, lies in the incomplete scattering of the spot itself.

ii. A modification of this, invented I believe by Sir W. Abney, gives much better results. A screen, AB , has a hole, CD , $\frac{1}{4}$ inch high by $\frac{1}{8}$ inch broad. On one side of this screen, a piece of

opal glass $\frac{1}{4}$ inch square is placed so that one half of it covers the hole in the screen. A movable pillar, P, is placed to cut off the light from S_2 from this opal, as shown in the figure. The opal is viewed in a mirror, M, by an eye placed at E. It will be

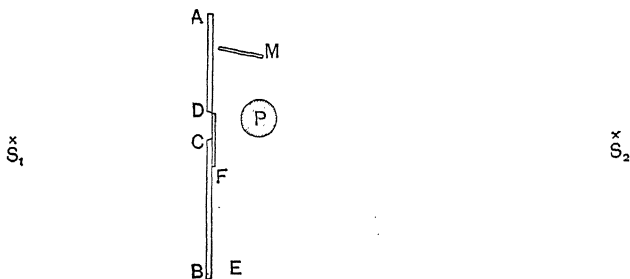


FIG. 107.—Improved Grease-Spot Photometer.

seen that the half CD of the opal is lighted by S_1 and CF, the other half, by S_2 . The thickness of the glass is reduced until the opal resembles a microscopic rock-section. The scattering is then very perfect, and very concordant readings are possible. The intensities of the lights will be proportional to the inverse square of their distances from the opal.

39. The Rumford Photometer.

Apparatus.—This consists of a board covered with black velvet upon which is mounted a small cube about 1 inch side, of which the front face only must be perfectly white. A movable vertical pillar about $\frac{3}{4}$ inch diameter is placed in front of the screen.

The two lights to be compared, S_1 and S_2 , are on the same side of the screen, and the pillar is adjusted until its shadows formed by S_1 and S_2 respectively just meet. The distances of S_1 and S_2 are adjusted, usually, by allowing one of them to move to and fro on a guide, until the two shadows are equally dark. When the pillar P is too close to the screen, a black line divides the shadows down the centre, and when too far away a bright line takes its place, but when the shadows just meet, there is no dividing line visible. The intensities of the lights will be proportional to the inverse square of their distances from the cube.

40. The Paraffin-wax Photometer.

This is one of the best of the simple photometers.

Apparatus.—Two pieces of paraffin wax, each about 1 inch square and $\frac{1}{4}$ inch thick, are carefully made of the same uniform thickness, a little over $\frac{1}{4}$ inch. A piece of tinfoil is placed between them, and they are fastened to the centre of a piece of wood or metal some 4 to 6 inches square. An aperture is cut in this $\frac{3}{8}$ inch square, as indicated by the dotted lines in Fig. 108, so that the edges of the blocks are visible through this aperture with the tinfoil as dividing line. The lights S_1 and S_2 to be compared.

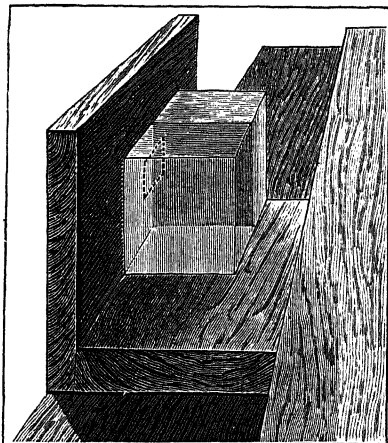


FIG. 108.—Paraffin-wax Photometer.

The lights to be compared are placed at the ends of a long guide, this photometer is moved to and fro upon the guide, until the halves of the paraffin wax seen through the aperture appear equally bright. The position is then read off on the scale attached to the base. It is an advantage to have



FIG. 109.—Paraffin-wax Photometer.

the photometer at the bottom of a block of cardboard or cloth box, that the eyes may not be dazzled by the direct light from the lamps. As an exercise, the light given by one wax

candle may be compared with that given by 1, 2, 3, 4, &c. The total length of this bench should be as great as can be conveniently arranged, so that when extended flames are being used, we may be able to consider them as points for purposes of calculation, and also that in comparing lights and their different intensity the distance from the weaker light to the screen may not have to be too small for measurement. It should be 12 to 20 feet long.

40a. The Lummer and Brodhun Photometer.

This depends upon a special prism which was invented by Prof. Wm Swan in 1859 and reinvented and, with the addition of the mirrors and screen described below, used for photometric work by Lummer and Brodhun. The prism is made of two right-angle prisms, of which the hypotenuse faces are cemented together to form it into a cubical block of glass. But a part of this interface is rendered reflecting, and it is here that the comparison of intensities takes place.

Apparatus.—Two right-angled isosceles prisms with faces say 1 in. to $1\frac{1}{2}$ in. square will be required (two of the prisms from a prism-binocular may be used); the reflecting area may most easily be produced in one of the following three simple ways:—

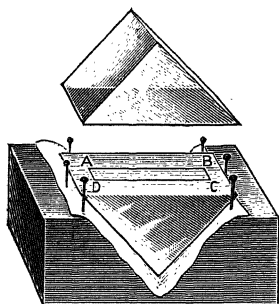


FIG. 109a.

(a) The hypotenuse of one prism is silvered, a strip about $\frac{1}{8}$ in. wide is painted over with warm wax, and the rest of the silver is dissolved off in nitric acid. The wax is removed by dipping it in ether or petrol; and then the two prisms are cemented together with canada balsam. To cement them, the two prisms, standing on a card,

are put in a rather cool oven; the balsam is also put in the oven. A right-angled V is cut in a block of wood (Fig. 109a); a piece of writing paper is put in the V and then one of the

of which a corner, HH, has been cut off. AB is a white screen (preferably made of plaster of Paris about $\frac{1}{8}$ in. thick, and say 2 in. square). CD and EF are two plane mirrors. (If the second or third method (*b* or *c* above) is used for combining the prisms, the light should make an angle, PON, less than 45° , to ensure that total reflection shall occur at the air space.)

The mirrors CD and EF should be so placed that the centre of the white screen AB is seen by reflection along the line PO. Upon the corner HH is mounted a tube, QR, in which a lens of about 3 in. or 4 in. focal length can slide, and be adjusted, so that the observer's eye at P may see the divided field at O distinctly. The box has apertures GG¹ and JJ¹ cut opposite to the screen AB, for the entry of the light. The box is mounted on a stand, and replaces the paraffin-wax photometer of Fig. 109. It will be much easier to determine the balance if the photometer is mounted on a carriage with wheels, so that it can be moved easily to and fro past the true position.

Experiment.—Set up the photometer between two lights which are to be compared, as was done in section 40. It will be found that the reflecting area (*i.e.*, in method *b*, the patch ABCD (Fig. 109*a*) is brighter than the surrounding area when the photometer is brought too close to the right-hand light; the reflecting area gets darker and the transmitting area brighter, as the photometer is moved towards the left. The position should be found at which both are equally bright). The distances from the lights to the white screen d_1 and d_2 are then to be measured; the intensities I_1 and I_2 will be inversely proportional to the squares of these distances.

$$\frac{I_1}{I_2} = \frac{d_2^2}{d_1^2}.$$

Interchange the two lights and repeat the experiment. This eliminates any want of symmetry in your apparatus. Take the geometric mean of your results as the final value.

40b. Polar Intensity Curve.

The brightness of the light given out by any lamp varies greatly in different directions, and it is interesting and often important to compare the intensities in directions making varying angles with the axis of the lamp.

Apparatus.—To determine this it is only necessary to mount the lamp on a turntable so that the direction of its axis can be varied, and also that the angle the axis has been turned through can be measured. The source itself (e.g. the filament if an electric lamp is being measured) must remain on the optic axis of the bench. A simple support which effects this is shown in Fig. 109c.

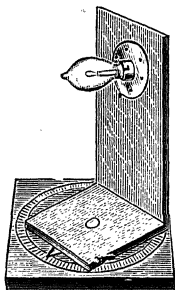


FIG. 109c.

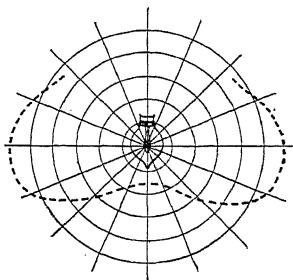


FIG. 109d.

Experiment.—Any convenient lamp can be used as a standard to compare this one with, its intensity being taken as unit. Then the intensity of the lamp under test is found for every $22\frac{1}{2}^\circ$ or 45° rotation of the axis, and these intensities can be plotted on a "polar diagram" (Fig. 109d), the angles being read off with a protractor and the intensities measured out from the centre along the radii.

40c. Mean Spherical Candle Power.

Definition and Units.—The intensity of the light given out by a lamp, as we have just seen, varies with the direction. For many purposes we desire to know the total flux of light emitted by the lamp. If we can find an experimental means of averaging the intensity of the light, we need only measure this average intensity, I_0 , in any direction, in order to determine the total flux.

Definition.—Let us call the amount of light falling on a unit area at unit distance from a light of one candle power, a *lumen*.

Then if I_0 is the average "intensity" of a light, I_0 is the average light energy that would fall on a unit area that is normal to the light and is at unit distance from the light; it is measured in lumens. The light falling on an area S , at a distance r , will then be $I_0 \frac{S}{r^2}$ lumens. On a sphere of radius r , it will be $I_0 \frac{4\pi r^2}{r^2} = 4\pi I_0$ lumens, = F the total flux or light energy emitted by the lamp.

Thus the total flux is 4π times the average intensity, and is measured in lumens.

"Definition.—The "lighting" "L" of a surface is measured by the amount of light falling upon a unit area. It is measured in lumens or in *lux*, calling a *lux* the flux of light from a lamp of one candle power falling on a sq. cm. at a distance of a metre, so that

$$L = \frac{I_0}{\left(\frac{r}{100}\right)^2} = 10^4 I_0 / r^2 \text{ lux.}$$

If the area is not normal to the incident light, but is inclined so that its normal makes an angle i with the direction of the light, the same amount of light has to cover a larger area, so the amount per unit area (*i.e.* the lighting) is reduced, and it is given by

$$L = 10^4 I_0 \cos i / r^2 \text{ lux.}^1$$

The best and simplest way to obtain the mean spherical candle power of a lamp is to put the lamp in a large white box (of any convenient shape, *e.g.*, a sphere, or a cube) which has one small window, and to measure the diffused light emerging from the window; care must be taken that the measuring apparatus

¹ The English lighting engineers usually measure the *lighting* of a surface in "foot-candles." A foot-candle is defined as the lighting which a lamp of one candle power would produce on a small area normal to the incident light placed at one foot from the lamp.

Then the lighting "L" of a small surface at a distance r from a lamp of candle power F , and so placed that the light falls normally upon it, will be given by

$$L = \frac{F}{r^2} \text{ foot candles.}$$

Or if the normal to the surface makes an angle i with the direction of the incident light the lighting will be given by

$$L = \frac{F \cos i}{r^2} \text{ foot candles.}$$

shall receive no direct light from the lamp. The best material for this enclosure is white oxide of magnesium, but plaster of Paris is nearly as good. White matt surface paper is also fairly good.

Apparatus (a).—A simple enclosure can be made of good white drawing paper. A sheet of this can be bent round to form a large cylinder, say a foot in diameter and 20 in. high; circular pieces of card are cut to form the top and bottom of the cylinder. The lamp is suspended from the top, and a hole about 2 in. in diameter is made in the side, well below the level of the lamp. This hole takes the place of one of the lamps in section 40, or 40a above.

Or (b) A cubical box of plaster of Paris is not very difficult to make. Begin by casting six slabs each say 18 in. square. These can be cast on glass; the glass is first painted with shellac varnish (shellac dissolved in methylated spirit) which will keep the plaster from sticking to the glass. The plaster should be mixed in a bowl with enough water to form a thick cream. A layer of plaster is poured on the glass, and pieces of wood, say 1 in. by 1 in. by 18 in., are placed round it. A piece of "scrim," or coarse canvas (sold at a paperhanger's warehouse), or other porous stuff, is dipped in the plaster and spread on the top of the plaster on the glass. More bits of wood are laid on the top of this stuff, to form ribs, and all is cemented together with more plaster. The body of the plaster need not be more than $\frac{1}{4}$ in. thick if it is stiffened in this manner. It is left an hour or so to set, and then taken off the glass. In this way the six slabs are made. After the hole for the lamp in one slab, and the hole from which the beam is to emerge in another slab, have been cut, the six slabs are joined together at the edges with plaster; the joint is strengthened by scrim soaked in plaster. The whitest plaster possible must be used. Such a plaster box will give excellent results.

Experiment.—The intensity of the light from the window is compared with that of the other lamp,

$$\frac{I_0}{I} = \frac{a^2}{a_0^2}$$

where a is the distance from the comparison lamp and a_0 is the distance of the hole in the box, from the photometer screen.

Then another lamp is substituted for the one in the enclosure, and the experiment repeated, obtaining a new average intensity I_0' .

$$\frac{I_0'}{I} = \frac{d'^2}{d_0^2}.$$

Thus we get I_0' in terms of I_0 . So if the candle power of the first lamp F is known, that of the second is also known, since

$$F = 4 I_0$$

and

$$F^1 = 4 I_0',$$

or

$$F^1 = F \frac{I_0'}{I_0}.$$

40d. Lighting. Lux-Meters.

Definition.—When work has to be done in artificial light, the most important measurement is the actual *lighting* of the surface or object. This, as already explained in section 40c, is measured in *Lux*, one lux being the quantity of light from a lamp of one candle power falling on unit area of the surface of a sphere of 1 m. radius

Apparatus.—It would obviously be very difficult to measure this *lighting* with any of the photometers so far described, for the measurement must be made *in situ*; and therefore a port-

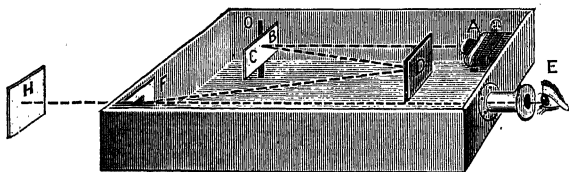


FIG. 1092.

able instrument is necessary. The simplest form of lux-meter depends upon the fact that the lighting of a surface upon which light is falling diminishes as the surface is turned with an increasing obliquity to the incident light. This must obviously be the case, since the same flux emerging from the lamp within any given solid angle has to be spread over an area which is a

minimum when the area is normal to the incident light, and becomes larger when the area is inclined to the direction of the incident light. An instrument making use of this principle can be easily made in a cigar box with the aid of an ordinary electric torch, or with a pea-lamp lighted by an accumulator. The torch A (Fig. 109*e*) lights a white card, BC, which can be rotated about an axis, O. The light from this card is reflected by a pair of plane silvered mirrors, D and F, to the eye E. The silver is removed from a small area in the middle of F through which the surface H, of which the lighting is required, can be seen. By rotating the screen BC, its brightness is varied, and is adjusted to equality with that of H. The position of the screen is read off on an arbitrary scale on the top of the box, by a pointer attached to the axis, O, on which the screen rotates.

The same principle can be still more simply applied to make a slightly different type of luxmeter. Let Fig. 109*c* be again supposed to represent a cigar box, with the eye-tube E fixed near the corner of one of the long sides. Cut a rectangular block of wood about $3'' \times 1'' \times 1''$, and pivot it at one end on a rod near the corner A, so that the other end can sweep round the arc of a circle from about C to D in that figure. In this end of the arm fix a pea-lamp; its light will then be made to fall on the box at F with varying obliquity according to the position of the arm, so that a screen at F would be brightly illuminated when the lamp is near the position of D in the figure, and will receive very little light when the lamp is in the position C. At F, in place of a mirror, put a narrow strip of white celluloid or paper, say $\frac{1}{8}''$ wide, along the vertical diameter of the aperture (which aperture may have a diameter of $\frac{1}{4}''$). The rod on which the arm is pivoted should be long enough to extend through either the top or the bottom of the box, and a pointer be attached to it outside the box, its position being read off on an arbitrary scale fixed to the box. To fix the arm and pointer to the rod, slit them lengthwise for an inch or so, the cut passing through the hole in which the rod is to go, and then put a screw through the arm transversely with a nut on its end, by which the slot can be squeezed together to grip the rod.

In both of these luxmeters a partition should be put between the path of the light from F to the eye, reaching far enough

towards F to prevent any direct light from the lamp from entering the eyes. Also the inside of the box should be blacked with a dull black, made by dissolving celluloid in amyl-acetate and adding gas black.

It must be remembered that the candle-power of any lamp varies very greatly with the voltage, and therefore it is important that the cell or accumulator used to light the little lamp in the luxmeter shall always have the same voltage when any measurements are being made.

Experiment.—A suitable exercise consists in the calibrating of the scale, by balancing it against a white paper at H, which is illuminated successively by 1, 2, 5, 10, 20, 30 lux; *i.e.* by 1, 2, 5, 10, 20, 30 candle power in turn, all at a distance of 1 metre, or by their equivalents (*e.g.*, 16 cp. at 4 m. is equivalent to 1 cp. at 1 m.). Thus actually the surface of a piece of matt white paper may be lighted by a 16 cp. lamp placed normal to the paper at say 50 cms., 70 cms., 1 m., 1.5 m., 2 m., 3 m., 4 m., from the paper. Calculate the "lighting" at each distance. Plot this lighting against the arbitrary scale readings; and so construct a graph, which will give the lighting corresponding to each scale reading.

As a further exercise, determine by actual experiment the lighting that appears desirable for such occupations as reading, drawing, white needlework, dark needlework, silversmith work, etc.

ADDITIONAL EXERCISES ON CHAPTER VII

1. Measure the brightness of an incandescent gas (or electric light) in terms of that of a wax candle, with each of the photometers, and compare the results.

2. Place sheets of white card, in the neighbourhood of each of the photometers in turn, so as to reflect the scattered light on to the instrument, and again measure the brightness of the incandescent light. Hence determine upon which photometer scattered light has the greatest effect.

3. Measure the candle-power of an electric lamp when the potential across the terminals, measured on a voltmeter, is varied by including a variable resistance in series with the lamp.

CHAPTER VIII

THE EYE AND VISION

41. Illustration of the Behaviour of the Muscles which move the Eye as a whole.

Apparatus.—Stereoscope ;¹ a stereoscopic picture ; pieces of ordinary red glass and green signal glass.

The axis of the eye may be pointed up or down, and may be moved right or left, the eye can also be rotated on its axis. The muscles which control the two eyes act together, as is well known. To a small extent they can be adjusted independently of one another.

i. Take two similar pictures : for instance, similar advertisements or concert tickets. Place one in the stereoscope opposite each eye, and adjust them until they are seen as one. It will be found that keeping one still, the other may be moved right or left for half an inch without any perceptible discomfort, and that coincidence may still be maintained at greater distances than this, but with a certain amount of pain. Do not attempt to carry it too far, as it is not advisable to run any risk of injuring the eyesight. In the same way it will be found that one of the pictures may be moved up and down to a slight extent and coincidence still maintained, showing that to a slight extent the muscle which moves one eye in that direction can be used independently of the other. As in ordinary circumstances we do not require to move the eye independently in this direction, we have not exercised these muscles independently, and it is probably for that reason that this motion

¹ A cheap form of this instrument is made, that will answer the purpose perfectly, costing less than 2s.

is so small. It is conceivable that with exercise it might be possible to make these motions quite independent.

ii. Lastly, keep one card at rest, it will be found that the other may be rotated slightly without becoming blurred. This can only mean that the corresponding eye has been rotated on its own axis to correspond.

Note.—These experiments can be shown to a class by using two lanterns and projecting two similar diagrams on the screen at the same time, one with red and the other with green glass, the members of the class being each supplied with a card having two holes the distance apart of the eyes, over one of which is a red and the other a green glass. By holding these in front of the eyes only a red picture will be seen by the eye which looks through the red glass, and similarly with the green. Then move the slide in one of the lanterns to and fro, up and down, and rotate it.

42. The Blind Spot.

That part of the retina where the optic nerve enters is insensitive to light, being devoid of the minute rods and cones necessary to vision. It is called the Blind Spot, and is situated a little nearer the nose than the optic axis.

Apparatus.—Cardboard, red wafers or drawing-pins.

This may be proved by sticking a red wafer or a drawing-pin in a piece of card and making a cross some 6 or 8 inches from it. Holding this in the right hand, with the wafer to the right of the cross and in the same horizontal line, and closing the left eye, it will be found that at a certain distance, if the



FIG. 110.

attention is fixed upon the cross, the wafer becomes invisible. By bringing the card nearer, or moving the wafer farther from the eye, it will disappear.

If the card be inverted, so that the wafer is to the left of the cross, and the experiment repeated with the left eye, the position can again be found at which the disappearance occurs. By fixing attention on the cross, the axis of the eye is pointed at it, and the light from the wafer, after passing through the lens, proceeds to a point on the retina to one side of the centre, at a certain distance this point coincides with the blind spot and the wafer becomes invisible.

43. Stereoscopic Vision.

i. If, without moving the head, first one eye then the other be closed, whilst objects at different distances are observed, it will be noticed that there is an apparent displacement of a near object relatively to a further one. For instance, if a position be taken up such that a lamp or a retort stand in the room is between the observer and the window, and the eyes be alternately closed, the lamp will appear relatively displaced to the window bars. The images, therefore, of objects at different distances do not exactly coincide on the retinas of the two eyes. If attention be fixed on an object at a certain distance, objects at other distances will be differently situated relatively to it. It is this displacement which we associate in our minds with "relief." An object appears in relief when this relative displacement is produced. If an object outside the window be observed in relation to the windows, it will be seen that its relative motion across the bars is in the opposite direction to that of an object in the room.

ii. By taking photographs of natural objects, either with two cameras side by side, or with a single camera which is moved a few inches laterally between the two exposures, two pictures are produced in which a similar relative displacement of the objects at the varying distances occurs; and if prints of these pictures can be seen, one with one eye and the other with the other eye, similar displacements are produced on the retina as are produced when the actual objects are seen, and there will be the feeling of relief.

The stereoscope is designed to produce this effect.

44. The Stereoscope.

Apparatus.—In the usual form of instrument a pair of prisms, with curved surfaces which both bend the rays and enlarge the image, are placed at the distance apart of the eyes. The prisms should have such an angle that the two pictures to be viewed placed side by side are caused apparently to coincide with one another. Owing to the fact that the eyes are not always the same distance apart, it is an advantage if the distance between these prismatic lenses can be slightly varied. If an ordinary 8-inch lens be cut in halves, and the halves

placed one in front of each eye with their thin edges towards one another, a pair of stereoscopic pictures, not too widely separated, can be viewed.

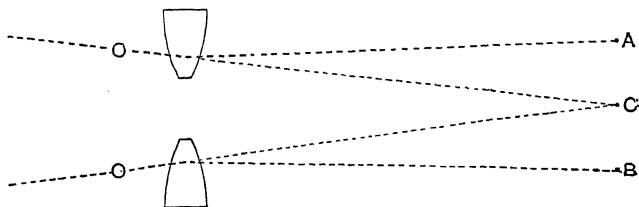


FIG. 111.—Principle of Stereoscope.

i. It will be easily seen from Fig. 111 that the light from the pictures A and B after passing through these two half lenses will appear to come from a point C.

When viewing an object at a short distance, the axes of the eyes are automatically converged to meet at that distance, so that to view these pictures at a distance of 8 or 10 inches, one adjusts the axes to meet at that distance; it is a strain upon the eyes to keep the axes parallel, and in viewing the pictures A and B, the axes will be pointed together and will meet at C. The distance apart of the pictures A and B, or the deviation of the rays by the lens, must be adjusted so that the light entering the eyes appears to come from C. As the lenses become more prismatic, the nearer the light passes through their edges, it is obvious that by altering the distance apart of the lenses so that the light entering the pupil of the eye (which is a pencil only 1 or 2 mm. in diameter) may pass through the lens nearer or farther from its edge, the amount of the convergence can be controlled. It is also obvious that, if the lenses are at such a distance that this convergence shall be correct for a person whose eyes are $2\frac{1}{4}$ inches apart, it would not remain correct for a person whose eyes are $2\frac{1}{2}$ inches apart.

ii. If a pair of simple stereoscopic pictures be drawn on two cards—for instance, one square within another, the inner square in one picture being to the right and in the other to the left of the centre of the larger square, and the corners of these squares connected as in Fig. 112, then if the cards are put in a stereoscope

so that one is viewed with the left eye and the other with the right eye, it is easy to see that the appearance must be that of a prismatic block, the inner square being nearer the observer than the outer. By interchanging the cards so that the left hand one is seen by the right eye, and *vice versa*, the appearance becomes that of a hollow tube.

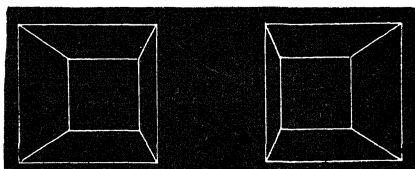


FIG. 112.—Stereoscopic Pair of Pictures of a Block. By holding a card vertically between the pictures, and adjusting the convergence of the eyes until the two appear superposed, an appearance of solidity is produced.

iii. If a stereoscopic photograph is taken of a piece of carving in relief, and viewed with the pictures correctly placed, the appearance of relief, of course, will be produced. But if the pictures are interchanged, so that the picture taken in the right-hand camera is viewed by the left eye, and *vice versa*, an appearance of intaglio will result. This reversed relief is known as *pseudoscopic*.

45. The Fovea Centralis.

The light after passing the crystalline lens and falling on the retina of an eye has first to pass a network of nerves and blood-vessels before it reaches the rods and cones. That is to say, the surface of the retina is placed the reverse of what would naturally be expected, the sensitive ends of the nerve fibres, the rods and cones, being below the nerves leading from them to the optic nerves as well as below the network of blood-vessels which feed them. But in the axis of the eye there is a spot which is free from this layer, and contains rods only and no cones, and for which the vision is much more acute than it is on the rest of the surface. That is to say, the defining power of this portion of the eye is greater. Its sensitiveness to light is not quite so good, and Abney has shown that its relative sensitiveness to the three primary colour sensations is not the same as that of the rest of the eye.

This spot may be observed if the light from a candle is reflected from the observer into another person's eye, who sits facing him at a short distance. If both observer and patient can fix their eyes at infinity, no lenses will be required, but as the observer expects to see this spot at

the distance of the retina, he only focusses his eye at that distance, and possibly the patient also focusses his eye at a short distance, and thus a clear image of the spot is impossible. In this case a concave lens will be needed.

Apparatus.—A special instrument called an ophthalmoscope is used by oculists for this examination, which constitutes an important branch of ophthalmic work. It consists of a concave mirror with an aperture in the centre.

The patient is placed with his back to a candle flame which is reflected by the mirror and focussed on the eye. A series of lenses fitted in a rotating diaphragm of varying focal

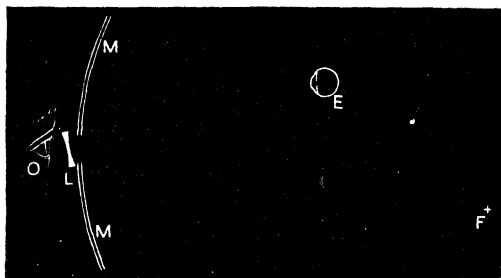


FIG. 113.—Examination of the Fovea Centralis of the Eye, E. F, candle; MM, concave mirror with a small central hole; L, lens; O, observer.

length can be brought into position over the hole, and the observer rotates this until he finds one to suit his sight. The back of the eye will then appear as a pink background traversed by blood-vessels and the centre spot is the Fovea Centralis.

The fact that the central spot is not so sensitive to light as the surrounding part of the retina has probably been noticed by every one watching a faint star on a clear night. If a star is so faint that it can hardly be seen, it will generally be seen much more easily if the attention is turned—that is to say, the axis of the eye directed, to a star in its neighbourhood. It has frequently been noticed that a poster which could not be read on the

opposite side of a railway station owing to insufficient light, could be made out by looking slightly to the side.

The increased distinctness of vision over this spot of the retina is a matter of everyday observation.

46. Chromatic Error.

The eye is by no means achromatic.

i. If a window is observed by the right eye, the left eye being closed, and a finger is moved across the eye from right to left, one side of each window-bar will appear distinctly coloured blue, and the opposite side yellow. If the finger is moved in the opposite direction, the colours will be reversed.

ii. If a spectrum is thrown upon a screen by a lantern using an achromatic lens, and the screen being square with the lens so that the width is uniform from end to end, the spectrum will always appear to get wider towards the violet end. If a good picture—a good diagram, for instance—is thrown upon the screen with a violet light, it will be found impossible to focus it when standing a few feet away; the violet light is converged so strongly by the eye, that it is impossible to accommodate the eye to view violet by it at any distance. A concave lens will at once allow it to be focussed sharply, showing that it was not due to insufficient light merely.

iii. If a distant object is viewed through a combination of coloured films which allows only the violet end of the spectrum to pass, it will be found that however brilliantly illuminated the object may be it cannot be distinctly seen.

iv. A red spot on a blue ground seems to be raised above the surface of the background, whilst a blue spot on a red ground seems depressed. This is due to the necessity for altering the curvatures of the crystalline lens in adjusting its focus for the red and blue respectively.

As we estimate distances partly by the convergence of the axes of the two eyes, that is stereoscopically, and partly by the curvature which we have to impress, by the ciliary muscle, upon the lens, it is obvious that the necessity for altering the focus of this lens must give the idea of a change in distance.

v. By projecting a spectrum on to a screen formed from a V slit so that the image on the screen consists of a brilliant V

having each limb drawn out horizontally into a spectrum, this image will seem to be in high relief, the red nearer and the violet farther from the observer.

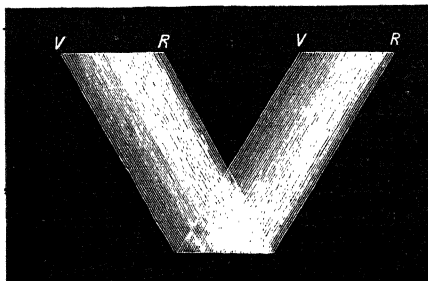


FIG. 114.—The spectrum formed from a V slit seems in relief.

vi. Look at the filament of an incandescent lamp through cobalt blue glass. This glass transmits both red and blue ends of the spectrum. When the observer is so near that he can focus the blue distinctly, but that his accommodation

is not sufficient to enable him to focus the red (the eye being, as these experiments show, of less power for red rays than for violet), he will find the filament appears blue with a red border. At a distance at which he can still focus the red, but at which his accommodation is too small to enable him also to focus the violet, the converse will appear. At a greater distance still, say 20 feet, where it becomes impossible even approximately to focus the violet, he will still be able to obtain a distinct, red image of the filament; but each point of the filament, through this error, will form a large diffusion circle of violet light; and these circles will be so large that the whole bulb will appear filled with a violet glow.

This experiment illustrates the fact that it is impossible at short distances, even, for instance, the distance at which we nominally place distinct vision—10 inches—to clearly focus an object in red light.

vii. If the filament is looked at through a deep red glass, this impossibility will be even more strongly illustrated.

Several other experiments illustrating this defect are given in Bidwell's *Curiosities of Light and Sight*. Foster's *Text-Book of Physiology* (Macmillan) may be consulted for experiments upon complementary colours, optical illusions, after-images, and other physiological matters in connection with the eye.

47. Images are seen inverted by the Eye.

Apparatus.—Thin card ; pins ; paper.

i. That images are seen inverted by the eye is most easily shown by holding a piece of card with a pin-hole in it at a distance of half an inch or an inch from the eye with one

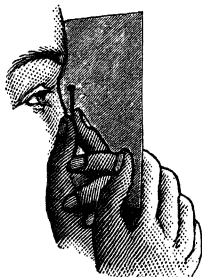


FIG. 115.—Inversion of Image on the Retina.

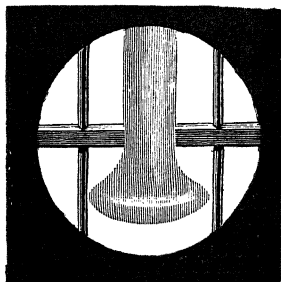


FIG. 116.—Window seen through Pin-hole, showing the shadow of the Pin, which seems inverted.

hand in front of a window, and passing a pin between the card and the eye with the other hand. The light from the hole in the card will spread on to the retina and the shadow of the pin will be cast upon the retina as it moves across. This shadow of course moves across the retina in the same direction as the pin, but it appears to move in the opposite direction. When the shadow of the pin is cast on the retina, if the pin be gradually moved until its head appears, the shadow will appear upside down. It must, of



FIG. 117.—Triangle of Pin-holes to be substituted for the Pin.

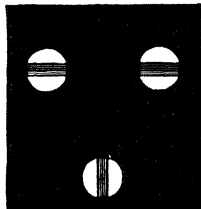


FIG. 118.—The Triangle formed by the three spots of light on the Retina seems inverted.

course, be erect on the retina. Thus, a shadow which is really erect on the retina appears inverted. Evidently then, as on

the ground glass of an ordinary camera, the images of the objects are always formed on the retina in an inverted position, and we have learnt to interpret them accordingly.

ii. The same experiment may be performed by making three pin-holes forming an equilateral triangle in a piece of paper, each side of which is about 1 mm. If this be held between the eye and the card with the single pin-hole, the light will be allowed to pass through in three rays and form three spots on the retina, which will be in the same relative position as the triangle itself. If, for instance, the triangle is held with its vertex uppermost, these spots will be similarly arranged. But they will appear inverted.

48. Accommodation—Long and Short Sight.

The greater part of the refraction in the eye takes place at the first entry of the light through the cornea. This outer surface can easily be seen to be more convex than the general surface of the eye by looking sideways at some one's eye. The light then passes through the iris, which has the power of expanding or contracting so as to keep the intensity of the light falling on the retina as nearly constant as possible, then through the crystalline lens. This lens is of the nature of a stiff jelly, and is naturally kept flat by muscles connected to its rim, and to the side of the eye. Surrounding the lens is a ciliary muscle, by the contraction of which the circumference is diminished and the lens caused to bulge, thus increasing its power. When looking at a near object, this increased power enables the more divergent wave from the near object to be sufficiently converged to focus on the retina. When the object observed is at a greater distance, the wave reaching the eye is flatter, and a lens of less power is needed to produce the right convergence still to focus it upon the retina. This is managed by relaxing the ciliary muscle.

When, therefore, a person is reading for any considerable time, this ciliary muscle is kept in a state of tension for that time, and if this is persisted in for many years, it is easily seen that the eyesight is likely to suffer.

As age advances, this muscle loses its power of contraction and the "accommodation" diminishes. By "accommodation" is understood this power of focussing the eye for objects at varying distances.

i. To Measure the Accommodation.

Apparatus.—Optical bench : convex lens of 10 inches focus : screen with small print on stand.

Place the eye *close* to a lens of about 10 inches focus, supported on the optical bench, and put a card, on which is some small print, in a stand. Look through the lens, and draw the card forward until the position at which it last ceases to be seen distinctly is attained. Note this position. Repeat the determination of this position some half a dozen times, withdrawing it from the lens and bringing it up again, so that the readings shall be as perfectly independent as possible, and take its mean position. Now withdraw the card to the greatest distance at which it can be distinctly seen, and find that position in the same way.

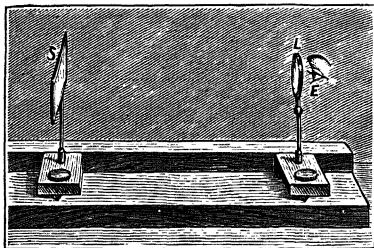


FIG. 119.—Accommodation.

Note the position of the lens, and having found the index error, obtain the two distances from the lens. Take the reciprocals of these in meters. The difference between these results is the accommodation.

ii. **Short Sight.**—If a short-sighted person tries this experiment, he will find that the two distances from the lens of nearest and furthest vision are each smaller than those found by a person of normal sight, but the accommodation will be the same.

It follows, therefore, that if a short-sighted person be given a lens which will make the furthest of these distances the same as the furthest distance for a normal-sighted person, he will have the same range of vision.

The farthest distance at which the card can be distinctly read by a normal-sighted person will be generally a little greater than the focal length of the lens. This shows that the eye is able to focus clearly not only the light from an object at an infinite distance, that is, a plane wave, but also to focus a wave that is slightly converging. If, however, a short-sighted person is given a glass that will enable him to focus a plane wave, it will be quite sufficient.

If the power of the lens used in the experiment is supposed to be 5 dioptries, and the reciprocal of the greatest distance of the card from the lens at which the short-sighted person could see it clearly were 7.5 dioptries, he would require a concave lens of 2.5 dioptries to make his sight normal.

iii. **Old Age.**—If an old man tries this experiment, he will find that the difference between the reciprocals of the distance of the card from the lens—that is, the difference between the curvature of the light entering his eye for farthest and nearest vision—is much smaller. The accommodation is less. No lens, therefore, will make his eye-sight entirely normal. He may require two lenses. (If his accommodation is half the normal accommodation, this would be sufficient to enable him to see at all distances.) His accommodation, however, will finally almost disappear, and then he will require a different lens for each different distance. Usually he will be content with two lenses, one to enable him to see distinctly at about 15 inches, for reading purposes, and another to see distinctly at about as many feet, for use out of doors.

iv. **Long Sight.**—Some persons suffer from the opposite defect to short sight—namely, hypermetropia. This must not be confused with the loss of accommodation caused by old age. With these patients, the distances for nearest and longest vision will be each longer than the distances for a person of normal eyesight. The accommodation will be the same. This error in vision may be perfectly corrected by the use of a convex lens of such a power that the longest distance of distinct vision is made normal; that is, the power of the lens must be the difference between the curvature of the light-wave from this greatest distance when it reaches the lens, and the power of the lens used in the experiment.

49. Astigmatism.

Nearly every eye is slightly astigmatic; that is to say, very few eyes indeed are perfectly symmetrical about their axes. This can be shown by taking a cylindrical lens of very low power, say 0.25 dioptries (see p. 53), and holding it before the eye, rotate it slowly while observing some distant small print. It will, perhaps, seem more indistinct in every position than when viewed with the naked eye alone, but as the

lens is rotated, in certain positions the indistinctness will be more marked than in others. Were the eye perfect, the orientation of this cylindrical lens could make no difference. If the print is distinctly clearer in one position of the lens than it is without the lens, a stronger lens of 0.75 dioptries should be tried. If with this lens it is more distinct than with the naked eye, especially if the student suffers at all from headache, there is no doubt that his eyes are seriously at fault, and he should consult a specialist without delay.

Astigmatism is due to the power of the lens being a little greater in one direction than in another. Thus, for instance, if the lens is slightly flatter in a horizontal section than in a vertical direction, vertical lines, which are focussed at a distance depending upon the horizontal section, will be focussed at a greater distance than horizontal ones. Both lines of a cross drawn on a card would not be focussed perfectly at the same time—but either the vertical or the horizontal line could be focussed at will.

Apparatus.—Convex lens of short focal length (say, six inches), stand for same ; white card with a series of parallel lines on it, and stand ; scale or optical bench.

i. **Examination for Astigmatism.**—If the convex lens is placed close in front of the eye (as in Fig. 119), and a well illuminated card on which are a series of horizontal lines is gradually approached to the lens, it will be found that at a certain distance it becomes impossible to retain the lines in focus. If the card be now turned through a right angle, so that the lines become vertical, and the distance found at which they again become indistinct (the eye throughout being kept close to the lens,) this distance will often be found slightly different from the previous one. Also, if the farthest distances be determined at which they can be clearly focussed, the distances will usually be different in the case of the horizontal and vertical lines respectively.

Take the reciprocals of these distances, and knowing the power (see p. 55) of the lens, determine the curvature of the light entering the eye in each of these four cases.

The difference between the curvatures when the horizontal lines are nearest and when they are farthest measures the accommodation of the eye.

Similarly, the difference of the curvatures when the vertical lines are nearest and farthest, measures the accommodation of the eye. These numbers should agree.

The difference between the curvatures of the horizontal and vertical waves measures the astigmatism in one direction, but is not necessarily the true astigmatism of the eye, because the greatest and least powers of the lens may not be horizontal and vertical, but may be oblique.

ii. **Measurement of Astigmatism.**—Placing the card at the greatest distance at which it is clear, rotate it and see if in any position it can be seen at a still greater distance; if so take this distance. Then turn it through a right angle and again find the greatest distance at which it is visible. The difference between the curvatures in these two positions will more approximately measure the astigmatism of the eye. Even this will not be accurate, as the eye possesses in its ciliary muscles, a power of adjusting the curvature not only as a whole but locally, and thus by these muscles it is able to slightly correct the natural astigmatism of the lens.

To find the astigmatism it is necessary, therefore, to paralyze these muscles with some drug in order to avoid the confusion caused by this means of accommodation. The correction of astigmatism by the ciliary muscles produces a strain upon those muscles which first causes headache, and if continued may permanently injure the eyesight.

Having found the difference between the curvatures, a lens the power of which in one direction is that amount, that is a cylindrical lens, will have to be added to the eye to increase its power in the required direction.

50. Colour Vision.

i. **Experiments illustrating Absorption of Light.**—(a) Make a solution of alkaline litmus and another of chromate of potash. These will be found with a spectroscope to remove the yellow and the blue, respectively, from white light. A mixture will therefore leave red and green. Or, the two solutions may be placed in flat vessels one behind the other. Pass white light through them, it will appear yellow. (A yellow disc can be projected on a screen by passing the light of a lantern through the solutions). Examine a sodium flame through the solutions one behind the other; it is invisible.

(b) Another yellow liquid similar in appearance can be made with a mixture of bichromate of potash, permanganate of potash,

and copper sulphate, or chloride; cutting out the green, blue, and red, respectively, and leaving a yellow. Look through this at a white light and at a sodium flame.

(c) Place this yellow liquid behind the other one,¹ and look through them. They are practically opaque.

(d) Dissolve cupric chloride in dilute hydrochloric acid; and also dye some amyl alcohol with an aniline red. Place these two liquids in a flat bottle and cork it up. The amyl alcohol will float on the top of the cupric chloride, and the two solutions will appear red and green respectively. Shake them up, and they will become black.

(e) Dissolve two parts of copper sulphate in twenty of water, and add ammonia until the precipitate is just dissolved. This gives a good blue solution, cutting off the red end of the spectrum. Make a solution of ferric sulpho-cyanide. This gives a good red solution. The two solutions, one in front of the other, produce black.

ii. **Recomposition of Light.**—(a) In front of a gas flame, or a piece of ground glass lighted by a window, or other white light, place a blue and yellow glass side by side, and examine them with a double image prism (p. 170). This will cause part of the blue and yellow to overlap. Where they overlap white light will result. (If two lanterns are available, blue and yellow patches may be caused to overlap, one from each lantern when again white will result.) If placed in front of one another, on a piece of ground glass, the blue and yellow frequently give green. A collodion film stained with methylene blue is better for this purpose than blue glass.

To produce this film, a piece of clean glass must be thoroughly dried; and a pool of pure collodion is poured on the middle of the glass, such as is sold for photographic purposes (the cheap collodion is opaque when

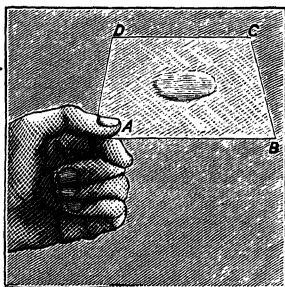


FIG. 120.

¹ Lord Rayleigh in *Nature*, January, 1871.

dry). The glass must be held by one corner between the thumb and finger of the left hand, as in (Fig. 120). By tilting the glass, the pool is caused to flow, firstly to the corner opposite C, then counterclock - wise round the plate to the corner D, then to A where the plate is held. Lastly, the excess is poured from the fourth and nearest corner B back into the bottle. The plate is slightly rocked while it is being poured off to avoid the collodion becoming streaked. In about a minute, before the collodion is thoroughly dry, it must be dipped in the staining solution (or the dyes may be mixed with the collodion).¹

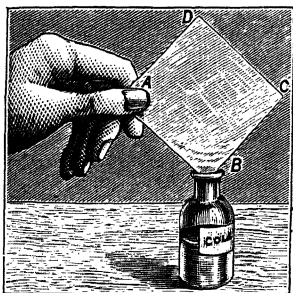


FIG. 121.

(b) Examine with the spectroscope the solutions above mentioned and explain the results. Also examine films dyed

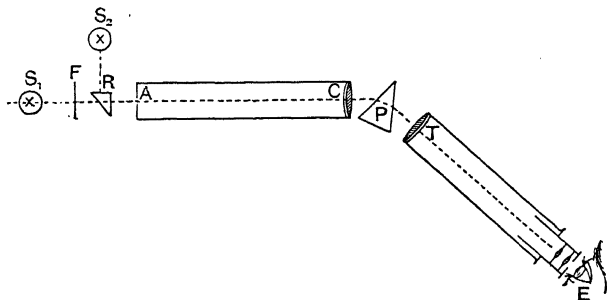


FIG. 122.—Examination of the Absorption produced by Coloured Films.

with the following aniline colours : magenta, rosaniline, rhodamine, eosine—which leave the red and part of the blue end of the spectrum ; chrysoidine, chrysophenine, cutting the blue end of the spectrum ; picric acid, which cuts the ultra violet ; and

¹ Instead of collodion, an ordinary unexposed gelatine photographic plate, that has been cleaned in "hypo," may be stained by immersion for several hours in an aqueous solution of the dye.

methylene blue, and malachite green, which cut the red end of the spectrum.¹

The films, or solutions, may be placed half covering the slit of the spectrometer, or the small right angled *comparison prism* may be adjusted in front of the slit (as in Fig. 122), so that two spectra, one of which has suffered absorption by the film and the other of which is left unaffected, may be seen the one above the other, and a curve drawn showing approximately the absorption of each solution.

Knowing the colours left by each film independently, it is easy to predict what will occur when they are bound two or three together ; and films may easily be arranged to give any colour effects that are desired.

¹ Small specimens in bottles of these dyes can be obtained of Messrs. Hartington Bros., 4 Oliver's Yard, City Road, at about 6d. each, containing enough dye for hundreds of films.

Stained gelatine films, such as are used on Christmas crackers, also make excellent colour absorption screens.

Messrs. Gallenkamp sell small bottles with worked flat sides, which are useful for introducing solutions in front of the slit of the spectrometer.

CHAPTER IX

FURTHER EXPERIMENTS WITH THE OPTICAL BENCH

51. The Refractive Index of a Concave Lens.

Apparatus.—Optical Bench ; bi-concave lens of about 12 inches focus.

- i. (a) Find the focal length by one of the methods already given.

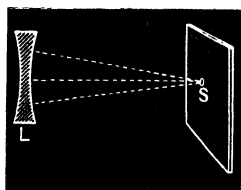


FIG. 123.—Curvature of the Surface of a Concave Lens, L. The light is reflected back and focussed on the screen, S.

- (b) Find the radius of curvature of each surface treated as a concave mirror by reflecting the light back to form an image of the ruled glass on the white paper round it. The distance from the surface to the screen is the radius of curvature.

Find μ , the refractive index, by substituting in the formula—

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{r} - \frac{1}{s} \right),$$

where f , r , s , are measured towards the light, and r is the surface upon which the light falls.

52. The Refractive Index of a Bi-convex Lens.

Apparatus.—Optical bench ; bi-convex lens of about 10 inches focus.

- i. (a) Find the focal length. This is most easily and accurately done by placing a plate-glass mirror behind the lens,

and reflecting the light back to form an image of the ruled screen on the paper round it. The distance of the lens from the screen is the focal length, f (with of course a negative sign).

(b) Move the lens forward and (without a mirror behind) obtain an image by the light reflected from the back surface of the lens. It will be much fainter than the one obtained with the mirror, but is easily seen. Let the distance of the lens from the screen be u_1 .

(c) Turn the lens round and again obtain an image by reflection from the back surface of the lens. Measure the distance of the lens from the screen. Let it be u_2 .

Then—

$$\mu = 1 - \frac{1}{2 + f \left(\frac{1}{u_1} + \frac{1}{u_2} \right)}.$$

Remember that f is negative.

In the case (b), the light is returning practically along its path, and therefore on reflection at the back surface, the incident and reflected rays must coincide, and must therefore strike the surface normally. So that the rays *in the glass* are radii of the surface; and if they emerged at the back of the lens,—as most of the light does emerge,—they will proceed without bending, since the incidence is normal; and will seem to come from the centre of curvature of that surface. But the point from which the emergent light appears to come is the “image.” Thus, in this case the image of the screen is at the centre of curvature of the back surface of the lens. As the focal length is known, we can easily calculate the position of this image by the formula—

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f},$$

and the v_1 so found is the radius of curvature of the back surface of the lens. Call it s_1 . Then—

$$\frac{1}{s_1} = \frac{1}{f} + \frac{1}{u_1}.$$

On turning the lens round, the curvature of the other surface is found in the same way from the formula—

$$\frac{1}{s_2} = \frac{1}{f} + \frac{1}{u_2}.$$

In the ordinary formula for a lens, $\frac{1}{f} = (\mu - 1) \left(\frac{1}{r} - \frac{1}{s} \right)$, the curvatures are to be measured towards the light, so that the curvature of the surface s_2 is to be considered negative. Substitute $r = -s_2$, $s = +s_1$,

$$\begin{aligned} \frac{1}{f} &= (\mu - 1) \left\{ -\frac{1}{f} - \frac{1}{u_2} - \frac{1}{f} - \frac{1}{u_1} \right\} \\ &= -(\mu - 1) \left\{ \frac{2}{f} + \frac{1}{u_2} + \frac{1}{u_1} \right\}, \end{aligned}$$

or—

$$\mu = 1 - \frac{1}{2 + f \left(\frac{1}{u_1} + \frac{1}{u_2} \right)}.$$

(Of course, f is a negative quantity as this is a convex lens.)

53. The Dispersive Power of a Convex Lens.

DEFINITION.—The dispersive power, ϖ , is the ratio of the dispersion, Δ , to the deviation, δ , therefore $\varpi = \frac{\Delta}{\delta}$.

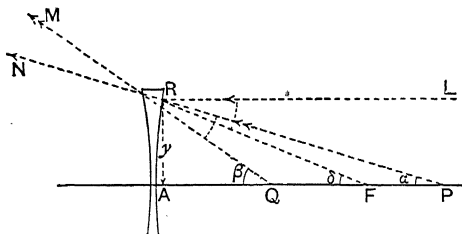


FIG. 124.

Let P and Q be conjugate foci of a thin lens, and F its principal focus. Then—

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

becomes—

$$\frac{1}{AQ} - \frac{1}{AP} = \frac{1}{AF}.$$

Call $AR = y$, and multiply this equation all through by y ,

$$\frac{y}{AQ} - \frac{y}{AP} = \frac{y}{AF},$$

or—

$$\beta - \alpha = \delta;$$

where α , β , δ are the angles made with the axis by PR,

QR, and FR, respectively; for, since the angles are small, we may use their circular measure instead of their sines, and write—

$$\beta = \sin \beta = \frac{y}{RQ} = \frac{y}{AQ} \text{ (nearly).}$$

But $\beta - \alpha$ is the angle QRP, *i.e.* the deviation. So that the formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ expresses the fact that the deviation of the ray from any point P, incident at a given point on the lens R, is constant, and equal to that of a parallel ray incident at the same point, and this deviation is $\frac{y}{f}$.

This deviation varies with the colour, and we have—

$$\delta_v = \frac{y}{f_v}, \quad \delta_r = \frac{y}{f_r},$$

where v and r refer to the violet and red light respectively. Thus—

$$\omega = \frac{\Delta}{\delta} = \frac{\delta_v - \delta_r}{\delta} = \frac{\frac{y}{f_v} - \frac{y}{f_r}}{\frac{y}{f}};$$

or, dividing by y ,

$$\omega = \frac{\frac{1}{f_v} - \frac{1}{f_r}}{\frac{1}{f}}.$$

The f in the denominator is supposed to be the focal length for yellow light.

Suppose the ruled glass, lens, and screen to be adjusted to give conjugate foci. The screen will have to be moved slightly to get the image clear for the different colours. If the lens and ruled glass are not moved, u will remain constant, and we shall have—

$$\omega = \frac{\left(\frac{1}{v_v} - \frac{1}{u}\right) - \left(\frac{1}{v_r} - \frac{1}{u}\right)}{\frac{1}{v} - \frac{1}{u}}$$

$$= \frac{\frac{1}{v_v} - \frac{1}{v_r}}{\frac{1}{v} - \frac{1}{u}} = \frac{\frac{v_r - v_v}{v_v v_r}}{\frac{u - v}{uv}};$$

or—

$$\omega = \frac{v_r - v_v}{u - v} \cdot \frac{u}{v}$$

(nearly, for v_v , v_r , and v may be considered equal for purposes of ratio).

Thus, we have to find the difference between v_u and v_r with the same accuracy as the other quantities involved.

Apparatus.—Optical bench : coloured glasses or solutions¹ ; positive eye-piece with cross-wires on stand, white light and a sodium flame.

(a) Adjust the cross-wires, lens, and focussing screen, and using a sodium flame find u and v as usual (p. 54) (correcting for index errors).

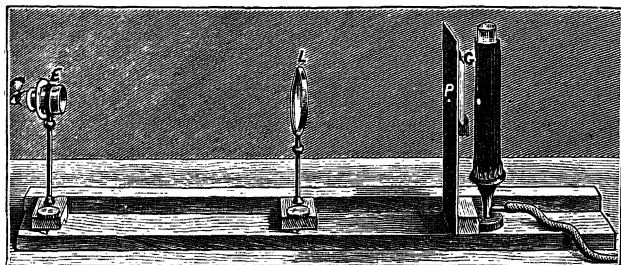


FIG. 125.—Achromatism of a Lens.

(b) Replace the focussing screen by the eye-piece. (See that the cross-wires are clearly in focus.) Look through the eye-piece, and the wires will appear very brilliant and highly coloured.

Place the red gelatine, G, close in front of the cross-wire screen, P (Fig. 125), and set the eye-piece so that the lines seem quite clear.

To obtain the exact position the method of parallax must be used. Adjust the eye-piece and the lens stands, until the cross-wire of the eye-piece is exactly on the image of the wires of the screen, and move the eye to and fro to see if they shift over one another, altering the distance of the eye-piece until there is no perceptible movement.

Then take the reading on the scale.

(c) Change the red gelatine for a deep blue one, and repeat the observation.

The difference between the readings with the red and the blue gelatines is $v_r - v_b$.

¹ See p. 138.

(d) Substitute in the formula—

$$\omega = \frac{v_r - v_u}{u - v} \cdot \frac{u}{v}.$$

(Note that v is *negative*).

The above determination does not give any definite value for ω , as we do not know for what wave-length we had adjusted the eye-piece. Much more accurate results can be obtained by using monochromatic lights.

A sodium flame should be used for the determination of u and v . The wave length is 5896. A hydrogen tube with a red glass and a blue glass may be used to find $v_r - v_v$. The lines given by the hydrogen tube are—a bright red line, C, of wave-length 6563; a greenish blue line, F, of wave-length 4863; and a bluish violet line, H, of wave-length 4341. (This last is very close to the Fraunhofer line G.) The red potassium line, K, 7677, gives another line in the extreme red, and by using it, the partial dispersion from A to D can be found if wished. (A is very near the potassium line.) The dispersion generally used is from C to F. As we shall be observing the image directly with an eye-piece, the small amount of light given by the hydrogen tube or the potassium flame will not matter, there will be sufficient for the purpose.

ADDITIONAL EXERCISES ON CHAPTER IX

1. Measure the focal lengths and the dispersive powers of each of the lenses of an achromatic combination.

Calculate $\frac{\omega_1}{f_1}$ and $\frac{\omega_2}{f_2}$. They should be numerically equal; for the condition for achromatism is $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$.

2. Repeat the determination of the dispersive powers with a different lens distance, and compare the results.

3. Examine the achromatic combination with the eye-piece, adjusted as in Experiment 53, and compare the appearance of the image with that formed by an ordinary simple lens.

4. Measure the curvatures of the surfaces of each of the lenses of the achromatic combination of Exercise 1, and calculate their refractive indices.

5. Find the ratio of the dispersive power of the glass, as found in the first two exercises, to the refractive index, in the case of each lens.

6. Find the refractive index, and the dispersive power of another lens. See if they are the same as those of either of the lenses of the achromatic combination.

CHAPTER X

INTERFERENCE AND DIFFRACTION

54. Interference.

IT is not very difficult to arrange a simple experiment to show the interference of light, the only piece of apparatus required that cannot be easily constructed being the bi-prism, and this can be obtained for a small sum.

Apparatus.—The bi-prism : slit : an eye-piece : supports for same. Cut a board about 1 foot long and 4 inches broad. Fasten to this board three upright blocks, each about 4 inches

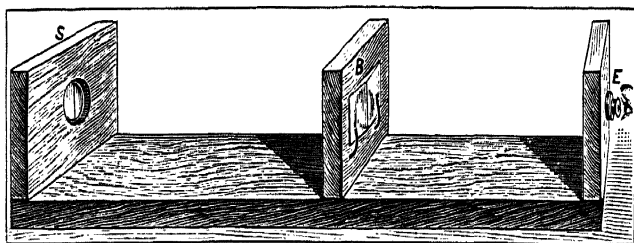


FIG. 126.—Simple Apparatus to observe Interference. A black cloth is to be tacked over it.

square with a hole about 1 inch in diameter through the centre of each ; the hole should just fit the eye-piece. Fix one of the blocks at each end of the board and one in the centre, with a single screw each, so that they may be rotated if desired, and also so that they may easily be placed at other distances from one another. Glue a piece of velvet or smooth-faced cloth to the face of the central block. For the slit two pieces of printers'

"rule" will do excellently, each about 2 inches long. Fasten one against the outside of the end block so that its edge is along the vertical diameter of the hole. Clip the other against the block under a spring of stout brass wire, it may then be adjusted to any desired distance from the fixed one, and the width of the slit varied. Clip the bi-prism in the same way against the central block under wire springs. A piece of black cloth should be cut to just cover in the space between the blocks, and tacked to the upper surface of each block, so that the hand may be inserted at either side to adjust the bi-prism.

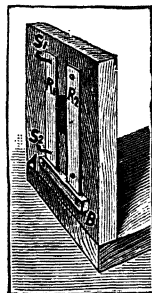


FIG. 127.—Slit.

Set the apparatus up opposite a good sodium flame, look through the centre of the hole in which the eye-piece is to be inserted and move the bi-prism sideways, until its ridge is in a line with the slit, when two images of the slit will be seen: one in each half of the bi-prism. On moving the head slightly to the right or left, one of the images will reach the ridge of the bi-prism and will be cut off. If the ridge of the bi-prism is exactly parallel to the slit (if, for instance, they are both exactly vertical), this image will be cut off along its whole length at the same instant; but, if they are not parallel, it will be cut off gradually, beginning, say, at the top. The bi-prism must be turned round until it is exactly parallel to the slit. Now insert the eye-piece, and the fringes should be seen. Very carefully rotate the bi-prism to and fro, and see if they can be improved; also see if by adjusting the width of the slit they can be improved. It will be found that if the slit is too wide they disappear, whilst, of course, if it is too narrow there will not be sufficient light to see them. The experiment should, if possible, be performed in a dark room. The sodium flame must be very bright to give good effects. Instead of the sodium flame, it may be pointed at an incandescent gas light, when the fringes due to white light will be seen. As the incandescent light is much brighter than the sodium flame, these are much more easily seen than the ones due to the sodium. They are, however, less numerous, only four or five being visible with white

light, whereas forty or fifty can be seen with the sodium flame.

The most important thing in this experiment is to adjust the bi-prism parallel to the slit. Unless this is done nothing but diffraction fringes will be visible.

To make the final adjustment of the bi-prism easily, some more sensitive means of rotating it is wanted. If a piece of

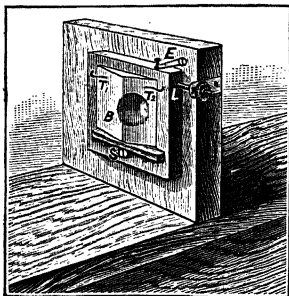


FIG. 128.—Mount for Bi-prism.

wood, somewhat smaller than the block, be fastened to its face by a single screw, S, as in Fig. 128, and the bi-prism attached to this, it can be rotated about this screw; and by attaching an ordinary levelling screw, L, to the side of the block, a very slow motion can be obtained, the wood being kept against the end of the levelling screw by an elastic band, E. The bi-prism can stand on a small ledge attached

to this block, being held in position by wire springs, T_1 , T_2 . If the hole in the block, that is to carry the slit, be made to fit the inner tube of the collimator, that slit can of course be used, though owing to the small height of the slit it will be at a sacrifice of light.

55. Diffraction Experiments.

Apparatus.—A similar piece of apparatus to that already described for the interference experiments can be used for the diffraction experiments, but with the distance, from the block carrying the slit to the central block, increased to 18 inches. No adjusting screw will be needed on the central block. The bi-prism is to be replaced by pieces of thin wood about $1\frac{1}{2}$ inch square and $\frac{1}{8}$ inch thick, each having a hole about $\frac{1}{2}$ inch in diameter through its centre. They should be blackened and fitted as follows:—

A. A piece of *printers' rule*, fastened vertically to the face with shellac varnish.

B. Break off $\frac{3}{4}$ inch of the points of four sewing needles of

different sizes, and attach these, at intervals of $\frac{1}{18}$ inch, vertically to the face so that the four points are on a horizontal diameter.

C, D, E. Three pieces of thin sheet zinc about 1 inch square with circular holes about $\frac{1}{20}$ inch, $\frac{1}{30}$ inch, $\frac{1}{100}$ inch in diameter. The holes must be drilled perfectly clean, preferably by twist drills, all burrs being carefully removed.

F. A flat spectacle glass is to be fastened against the face, and on the wood side are to be attached 4 or 5 leaden bullets of different

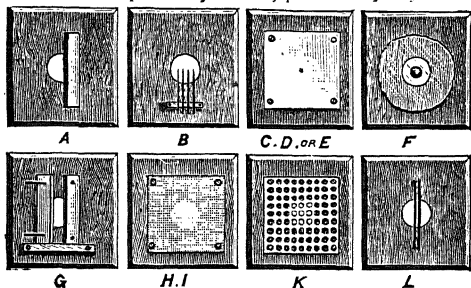


FIG. 129.—Objects for Diffraction.

diameters, from $\frac{1}{8}$ inch downwards. Care should be exercised in the selection of these that they may be as spherical as possible.

If bicycle balls of similar sizes can be procured, they may be substituted with advantage; they can be attached with a single drop of very thick shellac varnish, or by red wax; of course, the varnish must be entirely hidden by the balls.

G. A variable slit made as before with one fixed piece of brass rule, and another clipped by a wire spring.

H, I. Pieces of fine wire gauze of about 70 and 150 strands to the inch respectively.

K. A piece of perforated zinc.

L. Two pieces of thin knitting needle fixed parallel to one another.

For the end block, in addition to the slit (which will be used with numbers A, G, L, only), a circular aperture will be required. This can be drilled in a piece of zinc about 1 inch square, and should be about $\frac{1}{70}$ inch in diameter. It can be inserted under the wire spring, the slit being opened sufficiently to allow the light to pass.

Pointing the apparatus at a strong light, sketch the appearance seen in the eye-piece with each piece of apparatus, A—K.

If it can be pointed at the sun, the fringes will be far more brilliant.

When using the slit with numbers A, G, and L, care must be taken that the slit and the edges are accurately parallel to obtain the best effects.

56. The Bi-prism with the Spectrometer, used instead of an Optical Bench,

Apparatus.—An ordinary spectrometer with the table that usually carries the prism mounted on levelling screws; a bi-prism having a more obtuse angle than usual; ¹ sodium flame.

Remove the lenses from both collimator and telescope. Set up the bi-prism on the small levelling table, with its refracting edge coinciding with the axis of the instrument (*i.e.* over the centre of the table), and with its plane perpendicular to the line

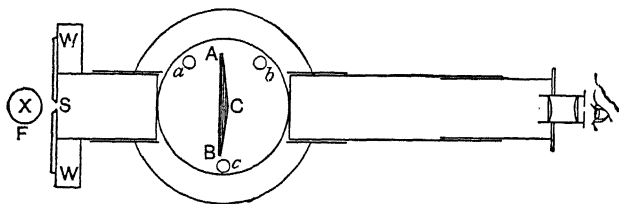


FIG. 130.—Spectrometer arranged to give Interference Fringes with a Bi-prism. The collimator has been removed, and replaced by a short tube carrying a wooden ring, W, W, on which a slit, S, of printers' rule is mounted. The rule is mounted like a parallel ruler, so that its width may be adjusted.

joining two of the screws. These two screws should be in the hole and slot respectively of the spectrometer table (p. 98). The third screw will then enable the bi-prism to be rotated slowly in its own plane, and thus allow the refracting edge to be set accurately parallel to the slit of the instrument.

Remove the eye-piece from the telescope, point it directly opposite the collimator, and look through it, when two images of the slit should be visible, one in each half of the bi-prism. By moving the eye, one of these can be caused to reach the refracting edge, and if the slit is parallel to this edge, it will

¹ The lines given by an ordinary bi-prism, so far from the slit as this has to be, are too fine, and generally cannot be seen at all; the ordinary bi-prism may be used, if the slit is brought near it, as in Fig. 130.

disappear from top to bottom at the same instant. If far out, rotate the collimator tube ; but, if nearly correct, adjust with the levelling screw of the prism table. Replace the eye-piece, cover up the centre of the instrument with a piece of black velvet, and the fringes should be visible. A slight adjustment of the levelling screw, and of the width of the slit will make them perfectly distinct. The distance apart of the bands will be equal to the distance, b , from the axis of rotation to the cross-wire, multiplied by the circular measure of the angle, θ , through which the telescope has to be moved to pass from one band to the next, and this can be easily determined. The distance apart of the image and the slit can be found by multiplying the distance, a , from the bi-prism to the slit, by the circular measure of the sum of the deviations produced by the halves of the bi-prism.

Thus, we require to measure—

- (1) The distance, a , from the bi-prism to the slit.
- (2) The distance, b , from the axis of the instrument (which must coincide with the edge of the bi-prism), to the cross-wire of the telescope.
- (3) The angular distance, θ , from line to line.
- (4) The angular distance, A , subtended at the edge of the bi-prism by the images formed in the halves of the bi-prism.

The first two of these measurements are straightforward though it is necessary to caution the student to be exceedingly careful in measuring the distance to the cross-wire of the telescope not to destroy it, as it is formed of two very delicate fibres.

To find the angular distance, θ , a series of 10 or 12 lines should be selected which are as clear as possible, and free from diffraction fringes which, it will be remembered, are superposed upon the interference lines. Set the cross-wire on the extreme line of the set selected, and take the reading of the vernier. Move the telescope, using the tangent screw so that the cross-wire is carried across the field, counting the lines as they pass. Again take the reading of the extreme line on the other side. The difference divided by the number of intervals will give the angular width of each.

To obtain the angular distance, A , subtended by the images formed in the halves of the bi-prism, the lenses must be replaced in both collimator and telescope, and the instrument focussed for parallel light, by the methods on p. 99. Then on pointing

the telescope at the collimator with the bi-prism in position, two images of the distant slit will be observed. Set the cross-wire on each of these in succession and take the two readings. The difference will be the angle A required.

It is proved in books on Physical Optics that if c be the distance between the two images of the slit, x be the linear distance apart of the interference fringes, and d the distance from the slit to the plane in which the fringes are produced, the wave length λ is given by

$$\lambda = \frac{cx}{d}.$$

In our case the distance apart of the images of the slit c becomes $a \cdot A \cdot \frac{\pi}{180}$, (since A will be given in degrees by the spectrometer scale),

x will be $b \cdot \theta \cdot \frac{\pi}{180}$, whilst d equals $a + b$.

Substituting we obtain—

$$\frac{abA\theta\pi^2}{(a+b)(180)^2}$$

As the interference fringes are very faint, owing to the small vertical height of the collimator slit, their distance apart is very difficult to obtain; for, if the instrument is well covered in so as to see the fringes with the greatest distinctness, the cross-wire is generally invisible; whilst, if sufficient scattered light be allowed to enter the telescope to illuminate the cross-wire, the fringes become invisible. By lifting and dropping the covering over the spectrometer table, the slit and the fringes can be seen alternately, and the cross-wire approximately adjusted upon a line. Of course, if the ordinary slit of the collimator be replaced by a temporary one of a vertical height as great as the diameter of the tube will allow, this difficulty will not be so great.

It is very important that the sodium flame should be as bright as possible. A good flame is produced by bending an iron wire of about number 22 B.W.G. into a circle about $\frac{1}{2}$ an inch in diameter, and coating the wire with a little asbestos, which is then dipped into a strong solution of salt in water. If this is placed over the top of a Bunsen burner so that the flame passes through the centre of the ring, it will everywhere be in the hottest part of the flame, and will give a powerful light which will last for a long time.

57. Lloyd's Mirror with the Spectrometer.

Apparatus.—Spectrometer; prism; sodium flame; white light (*e.g.* incandescent gas).

i. Remove the lenses of collimator and telescope, and place a prism on the levelling table, with one face across a diameter and parallel to the line joining two of the screws. Turn the table until this face is nearly parallel to the axis of the collimator. Remove the eye-piece, and, looking through the tube, turn the table until an image of the slit illuminated by the sodium flame is seen in this face of the prism, almost coincident with the slit itself. With the third levelling screw, adjust the face of the prism until this image is parallel to the slit.

Replace the eye-piece, and the interference fringes should be visible. They can be finally adjusted with the levelling screw of the table. Observe the effect of a slight rotation of the prism. As this will alter the distance from the slit to its image

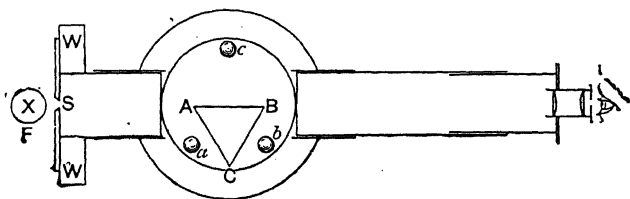


FIG. 131.—Spectrometer arranged to produce Lloyd's Fringes. A slit made of printers' rule has been substituted for the collimator as in Fig. 130.

as seen in the face of the prism, it will also alter the width of the interference fringes. The reading of the width of the bands may, if desired, be taken and treated as before. The distance of the slit from its image is more difficult to determine, as it depends upon the relative position of the face of the prism and the axis of the collimator. Thus, the insertion of the lenses will have to be made without disturbing either of these. This being done, the distance can be found and treated as before. As, however, these readings are very trying to the eyes, it is better to be content with the adjustment and observation of the fringes.

ii. Replace the sodium flame by white light, and insert a piece

of mica or micro-cover glass perpendicular to the reflecting face of the prism, so that the *direct* light from the slit has to pass through the mica on its way to the eye-piece. This will shift the centre of the system of fringes and enable it to be seen. Note that in white light the central band is *black*.

58. Diffraction with the Spectrometer.

Apparatus.—As in the last Experiment; the objects of Experiment 55, and an extra ring, W, W (Fig. 130), fitted with a pin-hole.

In the same way, the straight edge and narrow wire may be observed with the spectrometer, and their fringes sketched. Either of these diffraction experiments can be arranged as easily or even more conveniently than with the optical bench, as the instrument can be so easily covered in, a point of great importance.

Replace the slit by a pin-hole, either by opening its jaws and fixing a piece of tin foil over them with a hole in it; or, very much better, by withdrawing the tube carrying the slit, and replacing it by a tube with a pin-hole properly drilled in thin brass. Examine the circular aperture, opaque disc, and others of the set A to I on p. 149. All these diffraction experiments must be performed with the lenses of the telescope and of the collimator removed.

59. Herschel's Diffraction Experiments with the Telescope.

Apparatus.—Spectrometer with a pin hole in place of the slit of the collimator; circular stops of varying size to pull over the objective of the telescope; triangular apertures; perforated zinc.

A pin-hole mounted at the principal focus of the lens of the collimator is equivalent to a distant star and gives a beam of light which is parallel, not only in a vertical plane, as is the case with the slit, but is parallel in all aximuths. When a star is observed by a telescope, if the objective is a good one and well corrected for spherical aberration, the image will consist of a central bright disc surrounded by diffraction rings. The size of this disc, and the relative intensity of the disc and rings, depend upon the ratio of the diameter of the glass to its focal length. When the aperture is small the disc will be large, and the

brightness of the rings will also be comparable with that of the disc. As the aperture of the lens is increased, the disc and rings shrink and a greater proportion of the light is concentrated into the disc itself.

These effects can be easily observed by reducing the aperture of the telescope of the spectrometer with diaphragms containing circular holes of varying size. The collimator must be adjusted so that the circular hole shall be at this principal focus and the telescope focussed upon it. If the lenses of both collimator and telescope are good and the telescope is sharply focussed, even without any reduction of its aperture, a diffraction ring will be faintly visible, surrounding the image of the pin-hole. Now reduce the aperture by a diaphragm with a $\frac{1}{2}$ " hole. The diffraction ring will be much more clearly visible and will be larger, it will be quite circular. Use next an aperture $\frac{1}{4}$ " in diameter, then $\frac{1}{8}$, $\frac{1}{16}$ " and so on. The effect upon the disc and diffraction rings can thus be traced and the above statements verified. If other than circular apertures are used, the diffraction will vary accordingly; for instance, place a diaphragm with an aperture the shape of an equilateral triangle about $\frac{1}{4}$ " side over the lens of the telescope and a series of circles will appear in the form of a star with six arms.

A regular series of apertures such as those in a piece of perforated zinc will produce a strong diffraction effect. A series of irregular apertures punched at random will give circular diffraction patterns. These diaphragms can be made of metal or card, but the best material is *stout* "tin foil," i.e., thin sheet lead about No. 26, B.W.G.

ADDITIONAL EXERCISES ON CHAPTER X

1. Adjust the bi-prism as in Experiment 54, using an incandescent gas light (instead of adjusting them with a sodium flame, and afterwards substituting the white light).
2. Alter the position of the block carrying the bi-prism (Fig. 126), placing it about 4 inches from the one carrying the slit. Obtain the fringes, and see that they are much larger than before.
3. Remove the screw from the central block (Fig. 126), and insert the wire gauze object (Fig. 129). While looking through the eye-piece, gradually slide the block along the bench towards the slit. See that the fringes slowly close in.
4. In a dark room, look at a distant bright point of light (an arc,

if possible, or an incandescent gas light behind a small circular aperture), and hold first a hair, and then a straight edge, in front of the eye. Observe and draw the diffraction fringes.

5. Obtain a piece of old plate-glass, which has been scratched by repeated cleaning, and hold it near the eye while looking at a distant bright point. Diffraction fringes will be formed along lines which seem radial to the distant bright light. Explain these.

6. On withdrawing the eye, and fixing the attention upon the glass plate, bright lines will be seen which seem concentric with the distant point. Explain these.

7. Hold a small bunch of glass wool, or ordinary curly wool, near the eye, and observe it in the light of a distant arc, or the sun. A number of tiny circular coloured rings will be visible. Withdraw it a greater distance, and see that the wool forms numerous point images of the source of light, and that each of these point images when the wool is brought nearer, gives rise to a tiny circular system of rings. (These rings can frequently be seen on a sunny day in the loose nap of a woolly coat, or in the eyebrows.)

CHAPTER XI

NEWTON'S RINGS

60. Theory of the Method.

A. As the inclination of the surfaces used to produce the rings is very slight, where the rings are formed at any one point we may consider the two surfaces to be parallel.

Let AC and DB be the two surfaces separated by a layer of air of thickness e . A ray of light incident along PA will be refracted at A, reflected at B, and refracted at C, and emerge along CR. Another ray QC of the same parallel bundle as PA will also emerge along CR. Draw AM and CN at right angles to CQ and AB respectively.

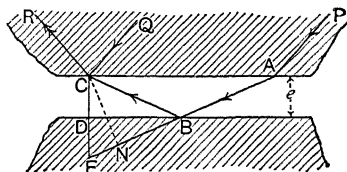


FIG. 132.

Draw CD at right angles to BD and produce to E making DE equal to CD. Then, as AB, BE are in the same straight line—

$$EN = EC \cos CEN = 2e \cos i.$$

Then the light will reach N from P, and C from Q together. Thus the difference of path at C is the difference between BC and BN; that is, EN, since BC = BE.

There is a difference of phase of half a period introduced, as one reflection (at C) is in glass from air, and the other (at B) is in air from glass.

Thus, the whole difference of path is $2e \cos i + \frac{1}{2} \lambda$.

If this difference is any whole number of wave lengths, the rings will be bright at that point. They are therefore—

bright if— $2e \cos i + \frac{1}{2}\lambda = n\lambda$.

dark if— $2e \cos i = n\lambda$ (1)



FIG. 133.

B. To get e in terms of the radii of the surfaces.

If both surfaces are convex the thickness e at any point is the sum of e_1 and e_2 , the amounts the surfaces are separated from the tangent plane at the point of contact.

Let x be the radius of the ring, i.e. = OM; R_1, R_2 , the radii of curvature of the two surfaces at O (Fig. 134).

Then as OM is a tangent—

$$OM^2 = MP \cdot MQ,$$

$$x^2 = e_1 \cdot 2R_1 \text{ (nearly),}$$

since MQ is nearly equal to the diameter of the sphere.

Or,

$$e_1 = \frac{x^2}{2R_1}.$$

So—

$$e_2 = \frac{x^2}{2R_2}.$$

$$\therefore e \equiv e_1 + e_2 = \frac{x^2}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right). \quad \dots \dots (2)$$

C. To calculate $\left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ from the spherometer readings.

When the spherometer (Fig. 137) stands on a sphere, the three legs lie on a small circle PS of radius ρ , and the central point touches the sphere at the pole of this circle, O (Fig. 135).

Let M be the centre of this circle and let OM = h (h is the reading of the spherometer). Then as PS is a chord we have—

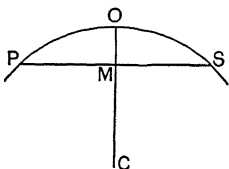


FIG. 135.

$$PM \cdot MS = OM(2R - OM),$$

or—

$$PM^2 = \rho^2 = h \cdot 2R - h^2 \\ = h \cdot 2R \text{ (nearly)}$$

(as in our case the curvature is very small) therefore—

$$\frac{1}{R} = \frac{2h}{\rho^2}.$$

To get ρ we may either measure the distance, a , between two of the legs, PQ, of the sphærometer, or we may measure it directly by screwing the central point down to the plane PQR, and then measuring OP (Fig. 136).

If we use the former—

$$\begin{aligned}\rho &= \frac{2}{3} a \cdot \cos 30^\circ \\ &= \frac{2}{3} a \cdot \frac{\sqrt{3}}{2} \\ \rho &= \frac{a}{\sqrt{3}}\end{aligned}$$

and—

$$\frac{1}{R} = \frac{6h}{a^2}.$$

Thus—

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{\rho^2} (h_1 + h_2); \dots \dots \dots (3)$$

or—

$$= \frac{6}{a^2} (h_1 + h_2).$$

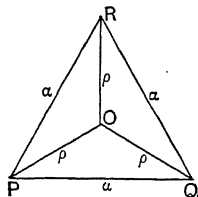


FIG. 136.

D. Then, if we take the dark rings—

$$\lambda = \frac{2e}{n} \cdot \cos i \dots \dots \dots \text{by (1)}$$

$$= \frac{2 \cdot \cos i}{n} \cdot \frac{x^2}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \dots \dots \dots \text{by (2)}$$

$$= \frac{2 \cos i}{n} \cdot \frac{x^2}{2} \cdot \frac{2}{\rho^2} (h_1 + h_2) \dots \dots \dots \text{by (3)}$$

$$= \frac{2 \cos i}{\rho^2} \cdot \frac{x^2}{n} (h_1 + h_2) \dots \dots \dots (4)$$

61. Newton's Rings by Transmission.

Apparatus.—A plano-convex lens of about 10 inch focus ; a flat glass surface (one face of a prism or cube, or a piece of plate glass, *not* a mirror) ; a reading microscope and sodium flame ; red wax ; silvered mirror ; blocks ; scale ; sphærometer.

Make two small pellets of red wax about the size of small shot. Clean the surfaces of the lens and plate. Holding them

parallel to one another with the curved surface of the lens facing the plate, blow smartly between them. Wait a moment for any condensed moisture to evaporate, put them together and place them on the table. Push the two pellets in between the lens and plate until they just touch both, forming a small equilateral triangle with the point of contact of the lens and plate. Press the lens down gently upon the pellets. This will cause them to adhere and the lens will now be firmly held, by its weight and the two pellets, upon the point of contact. The rings may be

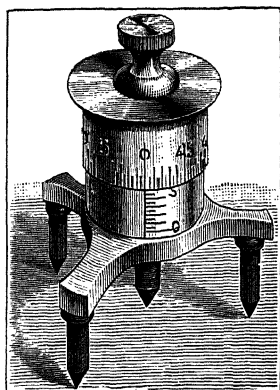


FIG. 137.—Spherometer.

viewed either by reflection or transmission. If by transmission, the system must be mounted on blocks and a mirror placed so as to reflect the light from the lamp vertically upwards through the lens. The reading microscope must then be pointed vertically down and adjusted so that it is focussed upon the system of rings. As the curvature of the lens is considerable, the system of rings will be very minute, and therefore rather difficult to find with the microscope. It can easily be seen with the naked eye as a little black speck if the

microscope is moved aside, and a small triangle of paper is laid on the lens with one corner pointing at the rings. If the microscope is focussed upon this corner of the triangle, and then slightly lowered, the rings will come into view.

When the rings are found they appear slightly hazy, and will now and then be seen suddenly to alter their position, and it is impossible to take good readings. This is due to the fact that the size and position of the rings depend upon the angle of incidence of the light. As the sodium flame has considerable area, if the rings are allowed to be lighted by the whole of this area, it is obvious that the angle of incidence is not definite, and the accidental variations in the brightness in different parts of the flame will determine the apparent position of the rings. It is

therefore necessary to limit the extent of the source of light. This can be easily done, if a hole about $\frac{1}{4}$ inch in diameter is cut in a piece of zinc, and after the rings have been found with the microscope, this is inserted near the flame between it and the mirror. By moving this diaphragm about, its best position can be determined. The rings will now appear perfectly sharp and steady. It is an advantage if the glass plate and reading microscope are mounted on a single slate slab isolated

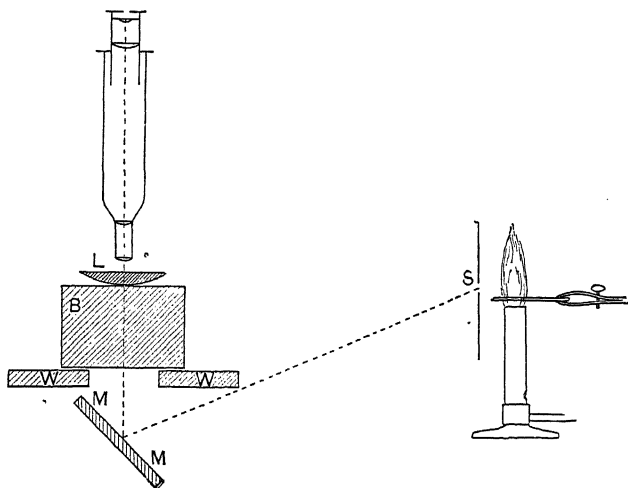


FIG. 138.—Newton's Rings by Transmitted Light.

L, lens ; B, glass block ; W, W, support ; M, M, silvered mirror ; S, diaphragm.

from the floor to avoid vibration, but this is not absolutely necessary. The readings must be taken from left to right only or from right to left only, to avoid the back lash of the screw.

The positions of the dark part of the rings can be entered in the following table, beginning with the tenth ring on the left, then reading the 9th, 8th . . . in succession across the centre to the . . . 8th, 9th, 10th rings on the right. These entries will thus be made in the second column of the following table from bottom to top, and in the third column from top to bottom.

No. of ring = n .	Left.	Right.	Diameter.	(Diam.) ²	$\frac{(\text{Diam.})^2}{n}$
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

The last column should be constant. One quarter of its mean value is to be substituted for the $\frac{x^2}{n}$ in the formula above.

62. Newton's Rings by Reflection.

Apparatus.—As in Exercise 61. To adjust the rings for reflected light, it is necessary to be able to tilt the reading microscope.

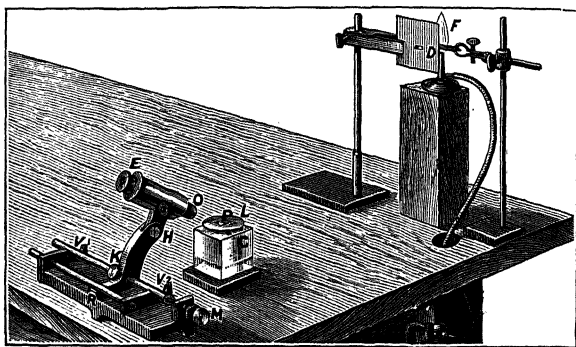


FIG. 139.—Newton's Rings by Reflection.

L, lens; G, glass block; E, eye-piece, and O, objective, of reading microscope; M, divided head of microscope; V₁, V₂, slots in which the rods carrying the microscope rests; K, clamping screw; the microscope is hinged at H to its support, and the support rests at R against the under-side of a fixed rod; F, flame; D, diaphragm.

Having attached the lens to the glass as already described, mount it horizontally on a block in such a position that the surface can be focussed with the reading microscope (this can be tested by placing a small piece of paper on the glass). Arrange a pointer opposite the eye-piece of the microscope. Remove the microscope without disturbing this pointer. Adjust the sodium flame, distant about 18 inches, so that its reflected image is seen by an eye placed at the tip of the pointer. In doing this, it will be found that at least three images of the flame are visible, one from the upper plane surface of the lens, one from the lower convex surface, and one from the plate glass beneath. (It is an advantage if the lens is tilted sufficiently to throw the first of these, which is much larger than either of the others, to one side, as we shall then only be dealing with the light reflected from the interfering surfaces, and shall be able to get perfectly black fringes.) The

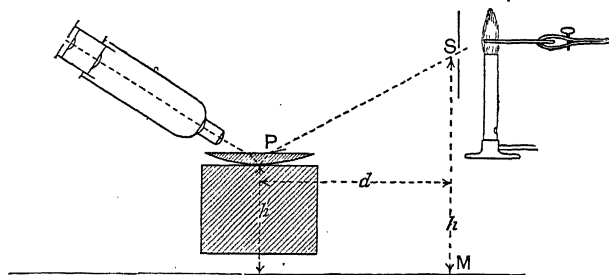


FIG. 140.—Newton's Rings by Reflected Light.

image from the convex surface of the lens will not coincide with that from the glass plate, unless the reflection takes place at the point of contact of the two surfaces, for only then is the tangent plane to the surface of the lens parallel to the plate. The flame must, therefore, be adjusted until these two images appear to coincide as seen from the pointer. When this is the case the small black dot which represents the Newton's rings system will be just visible. Place one corner of a small paper triangle, P, to appear to coincide with this spot as seen from the pointer. Replace the microscope, and focus it upon the corner of the paper triangle, when with a slight alteration of focus, the rings will be visible and a very small movement of the glass plate

will bring them into the centre of the field. The flame should be now slowly moved until the illumination is as powerful as possible, and finally the stop in the zinc plate must be inserted in front of the flame, and adjusted to produce the best possible effect, as described above for the transmitted system. The readings will be taken and tabulated as before. This time the angle of incidence will be wanted. It is best found from its *tangent*. Measure the height, h , of the slit above the table, and also h' , that of the system of rings.¹ Measure also the horizontal distance, d , from the system to the slit. Then—

$$\tan i = \frac{a}{h - h'}.$$

ADDITIONAL EXERCISES ON CHAPTER XI

1. Obtain the reflected system of rings, under the microscope (Fig. 139). Allow a drop of water to flow into the space between the lens and the glass block. It will be drawn in by capillary attraction. The rings will nearly disappear, but can still just be seen—especially if the light reflected from the upper surface of the lens has been eliminated by tilting the lens so as to reflect it to one side. The system will be so faint, that if once lost, it is practically impossible to recover it.

Note the reduced size of the rings, showing that the wave-length of light is less in water than in air. This experiment cannot be explained on the Emission Theory.

2. Place an ordinary plano-convex lens on a face of a highly refractive glass prism or plate. Focus the rings by reflection, under the microscope. The system should have a black centre. Without disturbing the apparatus, allow a single drop of oil of cedar to flow between the surfaces. The rings should now have a white centre, as the refractive index of cedar oil is intermediate between that of ordinary crown glass, and the highly dispersive glass prism.

3. The adjacent surfaces of the concave and convex components of an ordinary achromatic lens have a nearly equal curvature. Should the curvature of the convex surface be slightly greater than that of the concave surface, Newton's rings will be formed when they are placed in contact; and the system will have a black centre when seen by reflected light. Allow a drop of cedar oil to flow between the lenses: the centre will become white.

¹ It would be more accurate to adjust a piece of paper, P, on the lens, to just obscure the centre of the ring-system, and to measure h and d to this.

CHAPTER XII

POLARISED LIGHT

63. Experiments on the Polarisation of Light.

Apparatus.—The polariscope used in the following experiments may be easily made of wood. To a base board about

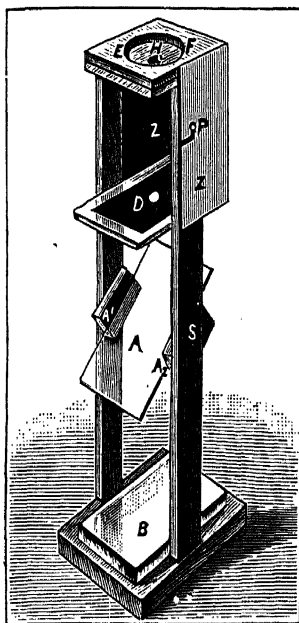


FIG. 141.—Norremberg Doubler.

6 inches square, attach two up-rights, each about 12 inches high and $\frac{3}{8}$ inch by 1 inch section. On the top of these fasten a stage, EF, also 6 inches square, having a hole $\frac{1}{2}$ inch in diameter at its centre. About 6 inches from the top, fix a horizontal glass plate, D, to act as a platform, and at about 4 inches from the bottom hinge another glass plate, A, about 5 inches square. An ordinary silvered mirror, B, about 4 or 5 inches square is placed on the bottom ; and a piece of zinc, Z, is bent round partly to inclose the space between the platform at the top and the glass stage. The zinc may be attached to the stand by a sort of bayonet joint, P, so

that it can easily be removed. Cut a hole about 3 inches in diameter in a thin piece of wood 6 inches square, and fasten this to the top of the table, EF. The glass plate, A, can easily

be hinged by inserting it in saw cuts made in small wooden blocks as indicated in Fig. 141. Cut a black card to lie on the glass stage, D, with a hole $\frac{1}{4}$ inch in diameter vertically under the hole in the top board. Draw lines on the top stage, EF, parallel to the sides, which would, if produced, pass through the centre of the circles, and also lines bisecting the angles between these, so as to mark every 45° .

Fitting easily in the 3 inch circle of the platform, EF, make three wooden discs each with a $\frac{1}{2}$ inch hole through its centre, of the same thickness as the board in which the 3 inch hole was made. To one of these, $E_1 F_1$ (Fig. 142), attach two uprights,

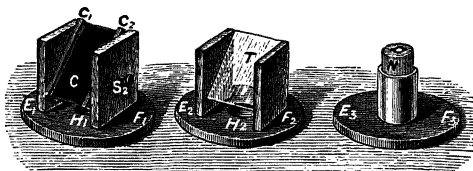


FIG. 142.—Fittings for Polariscopes.

and hinge between them in the same way as before, a small mirror, C (Fig. 142), about 1 inch by $1\frac{1}{2}$ inches, either of black glass, or of glass blackened at the back, with the reflecting surface downwards. On another disc, $E_2 F_2$, between two similar uprights, support a "pile of plates," T, of micro cover glass at an angle of about 53° . The glasses used by botanists, about 2 inches by 1 inch, answer perfectly for this purpose. To the third, $E_3 F_3$, attach a short tube which just fits a small Nicol's prism, N—one of those sold for attachment to the microscope is sufficiently large.

In addition to the polariscopes, two rhombs of Iceland spar of about $\frac{1}{2}$ inch side, a second Nicol, and a pair of tourmalines will be required; protractor, piece of spar cut perpendicular to the axis mounted in cork; tinfoil; two sewing needles stuck into a wood block with their points outwards and in the same vertical line.

i. **Use of Polariscopes.**—(a) Set up the polariscopes in front of a window so that the light from the sky¹ shall be—

1. Reflected vertically down from the inclined glass plate, A A (Fig. 143).

¹ At night a sheet of white card may be stood up in front of the instrument, with a good light illuminating it.

2. Reflected vertically up from the silvered mirror, B.

3. Partly transmitted by the glass plate, AA.

NOTE.—Nearly all the light is returned by the plate to the sky, but a small portion is transmitted.

4. Reflected by the black glass plate, C, into the eye.

(b) Without otherwise moving anything rotate the top black glass round a vertical axis, and observe the effect on the light.

In two positions the light will be nearly extinguished, and in two positions it will have maximum brightness.

Draw a diagram to indicate clearly the two positions in which it is dark, and the two in which it is bright.

Definition.—If plane polarised light be incident at the angle of polarisation on a glass surface (and the *plane* of incidence be varied by rotating the glass while the *angle* of incidence be kept the same), the **plane of incidence in which the light is best reflected is the plane of polarisation of the incident light.**

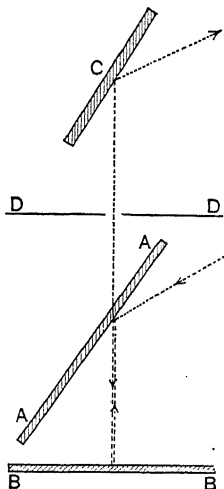


FIG. 143.—Path of the Rays through the Polariscope.

ii. **Plane of Polarisation.**—(a) Examine the light transmitted by the plate A, and using the above definition, find its plane of polarisation. Also state, with reasons, whether the polarisation produced by the plate A was produced by (1) the *reflection* down to B, or (2) the *transmission* through A.

(b) Replace the black glass mirror, C, by the pile of plates, draw diagrams as in Experiment 63, ii. (a), and compare them with those obtained in that experiment.

Show that the planes of polarisation of reflected and transmitted light are at right angles.

iii. **Brewster's Law.**—Replace the black glass mirror, and turn it to extinction. Now without rotating its plane alter its inclination, and that of the plate A, until the centre of the field is quite dark, and thus endeavour to find the angle of polarisation.

Measure this angle with a protractor ; and using the formula given by Brewster—

$$\tan i = \mu,$$

find the refractive index of the glass.

iv. **Double Refraction.**—(a) Examine the small rhomb of Iceland spar—there are eight angular points ; see that at six of these corners, there are two acute and one obtuse angle, but at the other two, which are diagonally opposite one another, there are *three* obtuse angles. *If the rhomb has all its edges equal* the

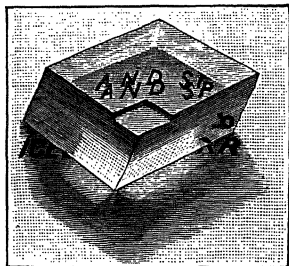


FIG. 144.—Iceland Spar.

line joining these two points is the *optic axis* of the crystal. Draw the crystal.

(b) Make a small black dot on a piece of paper, and place one of the rhombs upon it. You will see two dots. Draw a diagram of the crystal as seen from above, inserting in the drawing, the optic axis, and the line joining the dots. Notice carefully the relation of these two lines.

(c) Rotate the crystal and draw a diagram of the appearance every 90° . Which dot moves ?

(d) Raise the rhomb gradually from the paper, and notice carefully any change in position of the dots. Does their distance apart alter ? Hence show by diagrams in a vertical plane the course of the rays to the eye.

(e) Set up the block with the two needles so that their points are vertically over a black dot on a piece of paper (Fig. 145). Place the rhomb over the dot, and using the needles as a guide, observe which of the two dots coincides with the original dot. The dot that coincides is called the one formed by the ordinary ray, and the other is formed by the extraordinary ray.

(f) See that it is the ordinary ray that *remains still* when the crystal is rotated.

(g) Look carefully at the apparent positions of the two dots in the crystal. One will be seen to be higher than the other.

Which is it? Hence which refractive index is the greater—that of the ordinary ray or that of the extraordinary ray?

v. **Polarisation by Double Refraction.**—(a) Make a pin-hole in a piece of tinfoil, and lay it on the stage of the polariscope (Fig. 141). Place a spar rhomb on it, and looking directly down through the hole in the top stage, observe the effect of rotating the rhomb. Hence show that the light of each dot is polarised. Which ray is polarised in the plane containing the two dots?

(b) Place the rhomb on a black dot on a piece of white paper, and examine with the “pile of plates” the two dots produced. Remembering the plane of polarisation of the transmitted light, see whether your results agree with those obtained by the last method.

(c) Place a second rhomb over the first, and draw diagrams of the appearance of the dots at every rotation of 45° of the upper rhomb. In particular explain the positions of the *two* dots in each of the four positions in which only two are left.

(d) Test your explanation by examining these two dots with the pile of plates.

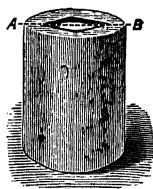


FIG. 146.—Nicol's Prism.

vii. **Nicol's Prism.**—(a) Place the *Nicol's prism* on the upper stage of the polariscope, and find the plane of polarisation of the light it transmits. Is it along the line AB?

(b) See that two Nicols produce more complete extinction than can be got with the reflecting polariscope.

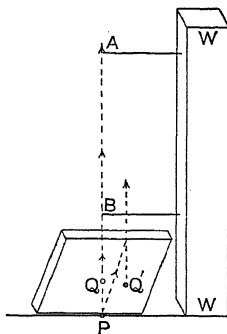


FIG. 145.—Discriminating between the Ordinary and Extraordinary Images.

viii. **Tourmalines.**—Examine the pair of *tourmalines*. Set them to allow the light to pass, and examine it with a Nicol. See that it is plane polarised. Note that the light they transmit is coloured. (Hence for most purposes they are inferior to a Nicol.) See that they produce very complete extinction.

64. Colours of Thin Plates.

Apparatus.—Polariscope ; mica and selenite films ; double-image prism ; Fox's wedge.

i. **Mica and Selenite Plates.**—(a) Set up the polariscope with a Nicol as analyser (*i.e.* in place of the pile of plates or the mirror C), and arrange it to extinguish the light. Also tilt the glass reflector, A, till the extinction is as complete as possible.

(b) Place a thin piece of *mica* or *selenite* on the stage, and note that colour is produced.

(c) Rotate the mica and see that in four positions at right angles it has no effect. In each of these four positions, rotate the Nicol, and see that the effect is precisely the same as if the mica were not present.

(d) Place the Nicol once more to extinction, and rotate the mica, note that the colour is brightest at angles half way between the positions at which it has no effect.

(e) Leave the mica in the position in which it has the greatest effect and rotate the Nicol. Notice that when it is turned 45° the mica once more has

no effect, on rotating another 45° the colour is again bright, but is the complementary of what it was at first.

ii. **Double Image Prism.**—(a) Replace the Nicol by the *double image prism* and see that the separate half images vary in the same way as those produced above, also that they are always

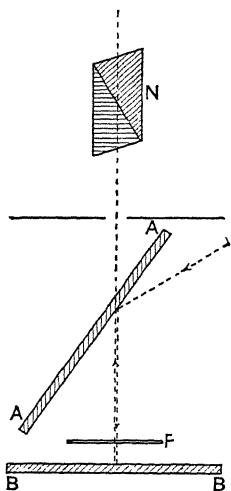


FIG. 147. —Polariscope arranged with a Nicol's prism, N.

complementary, and therefore where they overlap they form white. Note that the actual tint of the halves changes suddenly to its complementary, and that till that change they only vary by having more or less white mixed with them.

(b) Replace the mica by other pieces and note that the colour depends on the thickness.

iii. **Fox Wedge.**—Place the Fox wedge on the stage and note the colour changes. See that the colours produced proceed as in Newton's rings, and that in this wedge there are two orders of colour. (The wedges are made by cementing equal thicknesses of mica together so as to form a flight of steps each rising by an equal amount, see p. 177, Exercise 4.)



FIG. 148.—Method of arranging the Mica Films to form a Fox's Wedge.

In all the above cases plane polarised light falls on the crystal (mica) and is, in general, at once split into two beams, polarised in planes at right

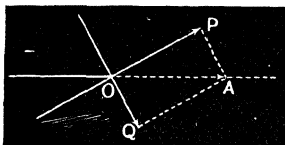


FIG. 149.—OA, amplitude and direction of a vibration incident on the crystal; OP, OQ, amplitudes and directions of vibration of the two components as they enter the crystal.

angles to one another, which may or may not be of equal intensity. These two rays travel through the crystal with different velocities, and, reaching the other side, have therefore gained or lost some fraction of a wave-length on one another. Here they recombine.

If either of the planes of polarisation of the light in the crystal happens to coincide with the plane of polarisation of the incident light,

of course there will be only *one* ray in the crystal. In this case, there can be no interference, whatever the thickness of the crystal. Hence, in four positions at right angles, the crystal has no effect as in Experiment 64, i. (c).

If the plane of the incident light *bisects* the angle between the planes of polarisation of the light in the crystal, as in Fig. 150, the two rays will be of *equal intensity*. In this case, the interference effects will be most marked; for should one ray of any colour gain half a wave-length, that colour will interfere completely.

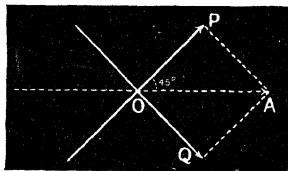


FIG. 150.—Component Vibrations with Equal Intensities.

If one ray gains any whole number of wave-lengths of any colour over the other ray, that colour will emerge in the same relative phase as it entered, and will combine therefore into a vibration in the same plane. So that, if the Nicols are crossed, that is, if they are arranged for extinction, that colour will *remain* dark: or, if the field had been bright, that colour will remain bright.

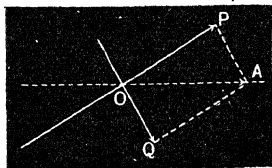


FIG. 151.—Decomposition of a ray at incidence on a crystal.

tion; on entry it is resolved into two components at right angles, OP and OQ. On emergence, if we suppose one component vibration to be in the same phase as at entry, to be, for instance, at Q' (Fig. 152), the other must be in the opposite phase to the one at entry; it will thus be at the other end of its swing at P'. The resultant of the component displacements OQ' and OP' will be the displacement OB. Also, as the component displacements at emergence will be along OQ' and OP', and will be equal in amount to those at incidence along OQ and OP, the resultant displacement will *always* be along OB. Therefore the resultant vibration will be along OB. That is, it is still plane polarised, but in a different plane. If, in addition, the angle AOP (Fig. 151) is 45° , it will be polarised in a plane at right angles, since the $\angle A'OB$ is double the $\angle A'OQ'$. Therefore, in a dark field, this colour (being rotated 90°) will appear bright, and dark in a bright field. Thus, we see why the colour depends on the thickness and why it changes to its complementary on rotating the analyser through a right angle.

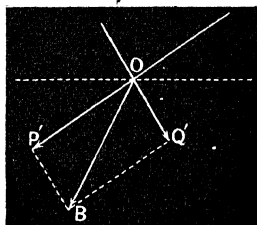


FIG. 152.—Recomposition of the light on emergence from a crystal when one component has gained half a wave-length on the other.

If one ray gains $\frac{1}{2}$ wave-length on the other for some colour, then Q will be in the middle of its swing when P is at the end of its swing, and *vice versa*. Hence, as they are at right angles and of equal amplitude, the resultant will be a circular vibration (Fig. 153). The colour will therefore appear equally bright with all positions of the

analyser. This is called **circularly polarised light**. (This can only be true, of course, for one colour at a time, and should be therefore observed with a sodium flame.) A film of mica of such a thickness that it produces this effect is called a *Quarter Wave Plate*.

If the gain is no exact wave-length, $\frac{1}{2}$ wave-length, or $\frac{3}{4}$ wave-length, the emergent light will have an elliptic vibration, and be called **elliptically polarised light**. Such a colour will increase and fade as the Nicol is rotated, but in no position can it be completely extin-

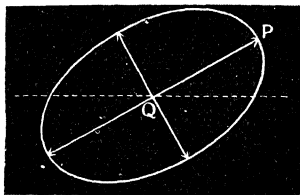


FIG. 154.—Elliptic vibration caused by a quarter wave plate, when the components have unequal amplitudes.

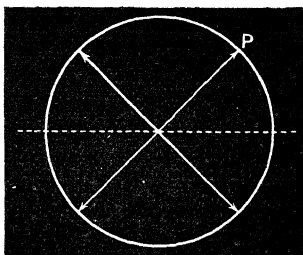


FIG. 153.—Recomposition of two components of equal amplitudes, on emergence from a crystal in which one has gained a quarter of a wave-length on the other.

guished.

If the gain were $\frac{1}{4}$ wave-length, but the axes in the mica were not at 45° with the plane of the incident light, the amplitudes of the two rays OP, OQ, would be unequal (Fig. 154). The effect would again be that the vibrations of the emergent light would be elliptic and not circular.

iv. Norremberg Doubler.—

Place the mica film (Fig. 147, F) on the mirror B. The colour will in general be quite different. The light now has to pass twice through the mica, and, therefore, as the ray which travelled most slowly on going *down* will again travel most slowly *after* reflection, it will lose *twice* as much as it would do were the mica on the stage D. The effect is thus the same as if a film of double thickness had been placed on D.

This form of polariscope is often called the Norremberg “doubler” in consequence of this action.

v. **Quarter Wave Plate.**—(a) Examine a quarter wave plate of mica on the stage D, using a sodium flame. On rotating the

analyser see that the intensity remains unchanged. [The mica must have its axes at 45° with the plane of polarisation.]

(*b*) Place the quarter wave plate on B, and see that it is bright in a dark field, and dark in a bright one, *i.e.* it acts as a half wave plate.

vi. **Half Wave Plate.**—(*a*) Place a half wave plate (*i.e.* a film of mica of a thickness that delays one component of the light half a period) on D, and see that it behaves as v. (*b*).

(*b*) Place it on B and note the effects.

vii. **Cause of Difference of Phase.**—(*a*) Take a small piece of mica of uniform thickness and scratch a line on it parallel to the direction in which it has no effect, cut it in two along some line (such as the dotted one). Place the pieces on one another with the scratch on each parallel and place it at 45° as usual. Note the colour when on D (in white light). Now place a single piece on B and see that you get the same colour.

(*b*) Take the same two pieces, but this time place them so that the scratch on one is perpendicular to the scratch on the other. Place the pair on either stage and see that it has no effect on the light in any position. Explain this.

65. Saccharimetry.

Apparatus.—A tube to contain the substance under examination about 20 cms. long, $\frac{1}{2}$ inch in diameter, and closed with worked glass ends—patent plate will do—(it will probably be simplest to procure this tube ready made) ; pair of Nicol's prisms,

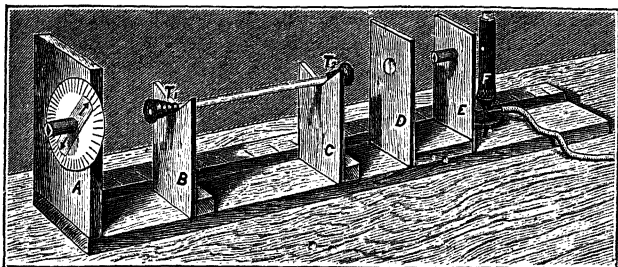


FIG. 155.—Saccharimeter.

which may be those supplied for use in a microscope, having a clear aperture of about $\frac{1}{8}$ to $\frac{3}{16}$ of an inch ; a biquartz ; these are to be mounted on a short optical bench similar to that on p. 46 ; beakers ; cane sugar ; a convex lens of about 4 inch focus ; an incandescent gas flame.

The tube is supported on a pair of V's. The biquartz is fitted in a hole in the stand D, and against this is the convex lens. One of the Nicol's prisms is supported in stand E behind the biquartz. At the end of this prism is a piece of card with a small aperture about $\frac{1}{2}$ cm. in diameter. A short brass tube, which will just fit the cork in which the other Nicol is mounted, is inserted in a hole in the stand A, in which it can be rotated smoothly. (If this hole is lined with cloth it will make the motion smoother.) A hole into which the brass tube will fit is cut in a piece of cardboard about 1 inch wide, and the tube pushed through it so that when the brass tube containing the Nicol is rotated in the hole in the stand A, this piece of card will be carried with it, and act as a pointer.

A divided circle, such as that ordinarily used for magnetometers, is attached to the stand A, and the card is cut just to reach the graduations of this circle. A pair of arrows are drawn on the card at opposite ends of a diameter. By always taking the readings of both arrows, errors of centering will be eliminated.

Make a 10 per cent. sugar solution—that is a solution containing 10 per cent. of sugar by weight and 90 of water. The solution must be perfectly transparent, and, if necessary, be filtered and left to settle. It must not only be free from dust but optically transparent ; that is to say, of uniform composition and therefore also of uniform refractive index.

The incandescent gas flame is placed behind E to illuminate the aperture, and the distance of D from E adjusted until the image of this aperture is formed at A. The tube for the sugar solution being removed the Nicol in A is rotated until the halves of the biquartz are of the same colour. The readings of both pointers are then taken.

Fill the tube with the sugar solution. If the tube has been washed in ordinary water, a film of water will remain inside the tube, and on putting in the sugar solution would mix with it and

alter the strength of the solution. Therefore a small quantity of sugar solution should be put in and well shaken up in the tube. A plug of cotton wool may be pushed backwards and forwards thoroughly to remove the film of water. Pour this away. Again partly fill the tube and notice if the solution remains transparent, or whether objects seen through it appear hazy. If so, the tube was not quite clean and must be further rinsed with sugar solution, this also being thrown away. Lastly, fill it right up, insert the tube between the stand A and the biquartz as in Fig. 155, and looking through the Nicol A, the halves of the biquartz will no longer be of the same tint. Rotate the Nicol in A until the equality is restored and obtain the reading.

Both this reading and the reading with the tube removed should be repeated several times, the mean of each being taken. The difference between the reading with and without the solution, of course, measures the rotation caused by the solution. As the rotation is very small the readings will have to be taken very carefully to obtain any trustworthy results at all.

The rotation is proportional to the percentage of sugar and also to the length of the tube. Thus, with the tube 30 cms. long, the rotation produced will be three times as great as would be given by a tube 10 cms. long, filled with the same strength solution. A twenty per cent. solution will give a rotation twice as great as a ten per cent. solution. Dividing the rotation produced by the percentage strength of your solution and by the length of your tube in centimetres, will give the rotation per centimetre of a one per cent. solution.

ADDITIONAL EXERCISES ON CHAPTER XII

1. Examine the light reflected from the surface of water with a Nicol's prism. It can be nearly extinguished at a certain angle of incidence, but not so perfectly as that reflected from a glass plate at the angle of polarisation.

2. Stand a board vertically in the water, and insert pins in it, to mark the direction in which the reflected ray is proceeding when the polarisation is most complete. The line, along which the water wets the board, will mark the surface of the water; therefore the angle of polarisation can be measured, and hence the refractive index of the water determined by Brewster's law (p. 168).

3. Examine with a Nicol the light reflected from a polished table, a shiny book, leather, and any other polished surfaces.

4. Make a Fox's wedge. Split off as large a sheet of mica as can be obtained.¹ Its thickness must be uniform, and as nearly as possible about $\frac{1}{8}$ -wave-length. This thickness will, when placed on the stage D, cause a dark field to become grey, but will give no colour. On the lower stage it should, of course, act as a $\frac{1}{4}$ -wave plate. (It is not absolutely necessary that it should be $\frac{1}{8}$ -wave.) Place it on the polariscope, and mark the direction in which it produces the most effect. Cut it into rectangles parallel to this direction, each rectangle being about $\frac{1}{2}$ inch in one direction, and in the other direction $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$ inch, and so on. These rectangles are now to be mounted upon a microscope slide, one upon another to form a flight of steps (Fig. 148), with Canada balsam dissolved in benzol or xylol. Leave them until the balsam is set; and then cover them with an ordinary thin glass, with more balsam. Be careful not to displace them in so doing.

¹ To split mica, insert a thin needle, perpendicular to an edge, about in the middle of one side, and then work it each way to split off the film so produced. By watching the *Newton's rings* which are seen where the films are separating, it is easy to see if the film is splitting properly. Any discontinuity forming in these rings, indicates a change in the thickness of the film. Before cutting the film, it must be examined with a polariscope, and if the thickness is not uniform all over, a scratch must be made surrounding the part which is of one thickness, and the rectangles must all be cut from this piece, so that each step in the wedge may increase by an equal amount. The wedge of Experiment 64 (iii.) is supposed to have sixteen films. The scratches will all disappear when the films are mounted in Canada balsam.

RECIPROCAL.

(Differences
to be subtracted.)

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	10000	9901	9804	9709	9615	9524	9434	9346	9259	9174	9	18	27	36	45	54	63	72	81
11	9091	9009	8928	8849	8772	8696	8621	8547	8475	8403	8	16	23	30	38	46	53	60	68
12	8333	8264	8197	8130	8064	8000	7936	7874	7812	7752	6	13	19	26	32	38	45	51	57
13	7692	7633	7576	7519	7463	7407	7353	7299	7246	7194	6	11	16	21	27	32	38	43	49
14	7143	7092	7042	6993	6944	6897	6850	6803	6757	6711	5	10	15	19	24	29	34	38	43
15	6667	6623	6579	6536	6493	6452	6410	6369	6329	6289	5	9	13	17	21	25	29	34	38
16	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	8	11	15	19	22	26	30	34
17	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	6	10	13	16	19	23	26	29
18	5556	5525	5494	5464	5435	5405	5376	5347	5319	5291	3	6	9	12	15	17	20	23	26
19	5263	5236	5208	5181	5155	5128	5102	5076	5050	5025	3	5	8	11	13	16	18	21	24
20	5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	9	12	14	17	19	21
21	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	6	9	11	13	15	17	19
22	4545	4525	4504	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	12	14	16	18
23	4348	4329	4310	4292	4273	4255	4237	4219	4202	4184	2	4	6	7	9	11	13	15	16
24	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	3	5	7	8	10	12	13	15
25	4000	3984	3968	3953	3937	3922	3906	3891	3876	3861	2	3	5	6	8	9	11	12	13
26	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	2	3	4	6	7	9	10	11	13
27	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	3	4	5	7	8	9	11	12
28	3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1	2	4	5	6	7	9	10	11
29	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	2	3	5	6	7	8	9	10
30	3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1	2	3	4	5	6	8	9	10
31	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	2	3	4	5	6	7	8	9
32	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
33	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	5	6	7	8	9
34	2941	2933	2924	2916	2907	2899	2890	2882	2874	2865	1	2	3	3	4	5	6	7	8
35	2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1	2	2	3	4	5	6	6	7
36	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	1	2	3	4	4	5	6	7
37	2702	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	1	2	3	3	4	5	6	6
38	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1	1	2	3	3	4	5	5	6
39	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	1	2	3	3	4	5	5	6
40	2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	6
41	2439	2433	2427	2421	2415	2410	2404	2398	2392	2386	1	1	2	2	3	3	4	5	5
42	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	4	5
43	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1	1	2	2	3	3	4	4	5
45	2222	2217	2212	2207	2203	2198	2193	2188	2183	2179	0	1	2	2	2	3	3	4	4
46	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	1	1	2	2	3	3	4	4
47	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	3	3	4	4
48	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0	1	1	2	2	2	3	3	4
49	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	2	2	2	3	3	4
51	1961	1957	1953	1949	1946	1942	1938	1934	1931	1927	0	1	1	2	2	2	3	3	4
52	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0	1	1	1	2	2	3	3	3
53	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	3	3	3
54	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0	1	1	1	2	2	3	3	3

RECIPROCAL.

(Differences
to be subtracted.)

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0	1	1	1	2	2	2	3	3
56	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0	1	1	1	2	2	2	3	3
57	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	2	2	2	2	3
58	1724	1721	1718	1715	1712	1709	1706	1704	1701	1698	0	1	1	1	1	2	2	2	3
59	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0	1	1	1	1	2	2	2	3
60	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	1	2	2	2	2
61	1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
62	1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0	1	1	1	1	1	2	2	2
64	1562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	1	1	1	1	1	2	2	2
65	1538	1536	1533	1531	1529	1527	1524	1522	1520	1517	0	1	1	1	1	1	2	2	2
66	1515	1513	1511	1508	1506	1504	1501	1499	1497	1495	0	1	1	1	1	1	2	2	2
67	1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	1	1	1	1	1	2	2	2
68	1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	1	1	1	1	1	1	2	2
69	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	1	1	1	1	1	1	2	2
70	1429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	1	1	1	1	1	1	2	2
71	1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0	1	1	1	1	1	1	2	2
72	1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	1	1	1	1	1	1	2	2
73	1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	1	1	1	1	1	1	1	2
74	1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	1	1	1	1	1	1	1	2
75	1333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0	1	1	1	1	1	1	1	2
76	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	1	1	1	1	1	1	1	2
77	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	1	1	1	1	1	1	1	1
78	1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	1	1	1	1	1	1	1	1
79	1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	1	1	1	1	1	1	1	1
80	1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	1	1	1	1	1	1	1	1
81	1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	1	1	1	1	1	1	1	1
82	1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	1	1	1	1	1	1	1	1
83	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0	1	1	1	1	1	1	1	1
84	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	1	1	1	1	1	1	1	1
85	1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	1	1	1	1	1	1	1	1
86	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	1	1	1	1	1	1	1	1
87	1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	1	1	1	1	1	1	1	1
88	1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	1	1	1	1	1	1	1	1
89	1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	1	1	1	1	1	1	1	1
90	1111	1110	1109	1107	1106	1105	1104	1102	1101	1100	0	1	1	1	1	1	1	1	1
91	1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	1	1	1	1	1	1	1	1
92	1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	1	1	1	1	1	1	1	1
93	1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	1	1	1	1	1	1	1	1
94	1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	1	1	1	1	1	1	1	1
95	1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	1	1	1	1	1	1	1	1
96	1042	1041	1040	1038	1037	1036	1035	1034	1033	1032	0	1	1	1	1	1	1	1	1
97	1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0	1	1	1	1	1	1	1	1
98	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	1	1	1	1	1	1	1	1
99	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	1	1	1	1	1	1	1	1

NATURAL SINES.

	0'	10'	20'	30'	40'	50'	1	2	3	4	5	6	7	8	9
0°	0000	0020	0058	0087	0116	0145	3	6	9	12	15	17	20	23	26
1	0175	0204	0233	0262	0291	0320	3	6	9	12	15	17	20	23	26
2	0349	0378	0407	0436	0465	0494	3	6	9	12	15	17	20	23	26
3	0523	0552	0581	0610	0640	0669	3	6	9	12	15	17	20	23	26
4	0698	0727	0756	0785	0814	0843	3	6	9	12	15	17	20	23	26
5	0872	0901	0929	0958	0987	1016	3	6	9	12	14	17	20	23	26
6	1045	1074	1103	1132	1161	1190	3	6	9	12	14	17	20	23	26
7	1219	1248	1276	1305	1334	1363	3	6	9	12	14	17	20	23	26
8	1392	1421	1449	1478	1507	1536	3	6	9	12	14	17	20	23	26
9	1564	1593	1622	1650	1679	1708	3	6	9	12	14	17	20	23	26
10	1736	1765	1794	1822	1851	1880	3	6	9	12	14	17	20	23	26
11	1908	1937	1965	1994	2022	2051	3	6	9	11	14	17	20	23	26
12	2079	2108	2136	2164	2193	2221	3	6	9	11	14	17	20	23	26
13	2250	2278	2306	2334	2363	2391	3	6	8	11	14	17	20	23	25
14	2419	2447	2476	2504	2532	2560	3	6	8	11	14	17	20	23	25
15	2588	2616	2644	2672	2700	2728	3	6	8	11	14	17	20	22	25
16	2756	2784	2812	2840	2868	2896	3	6	8	11	14	17	20	22	25
17	2924	2952	2979	3007	3035	3062	3	6	8	11	14	17	19	22	25
18	3090	3118	3145	3173	3201	3228	3	6	8	11	14	17	19	22	25
19	3256	3283	3311	3338	3365	3393	3	5	8	11	14	16	19	22	25
20	3420	3448	3475	3502	3529	3557	3	5	8	11	14	16	19	22	25
21	3584	3611	3638	3665	3692	3719	3	5	8	11	14	16	19	22	24
22	3746	3773	3800	3827	3854	3881	3	5	8	11	14	16	19	21	24
23	3907	3934	3961	3987	4014	4041	3	5	8	11	14	16	19	21	24
24	4067	4094	4120	4147	4173	4200	3	5	8	11	13	16	19	21	24
25	4226	4253	4279	4305	4331	4358	3	5	8	11	13	16	18	21	24
26	4384	4410	4436	4462	4488	4514	3	5	8	10	13	16	18	21	23
27	4540	4566	4592	4617	4643	4669	3	5	8	10	13	15	18	21	23
28	4695	4720	4746	4772	4797	4823	3	5	8	10	13	15	18	20	23
29	4848	4874	4899	4924	4950	4975	3	5	8	10	13	15	18	20	23
30	5000	5025	5050	5075	5100	5125	3	5	8	10	13	15	18	20	23
31	5150	5175	5200	5225	5250	5275	2	5	7	10	12	15	17	20	22
32	5299	5324	5348	5373	5398	5422	2	5	7	10	12	15	17	20	22
33	5446	5471	5495	5519	5544	5568	2	5	7	10	12	15	17	19	22
34	5592	5616	5640	5664	5688	5712	2	5	7	10	12	14	17	19	22
35	5736	5760	5783	5807	5831	5854	2	5	7	10	12	14	17	19	21
36	5878	5901	5925	5948	5972	5995	2	5	7	9	12	14	16	19	21
37	6018	6041	6065	6088	6111	6134	2	5	7	9	12	14	16	18	21
38	6157	6180	6202	6225	6248	6271	2	5	7	9	11	14	16	18	20
39	6293	6316	6338	6361	6383	6406	2	4	7	9	11	13	16	18	20
40	6428	6450	6472	6494	6517	6539	2	4	7	9	11	13	15	18	20
41	6561	6583	6604	6626	6648	6670	2	4	7	9	11	13	15	17	20
42	6691	6713	6734	6756	6777	6799	2	4	6	9	11	13	15	17	19
43	6820	6841	6862	6884	6905	6926	2	4	6	8	11	13	15	17	19
44	6947	6967	6988	7009	7030	7050	2	4	6	8	10	12	15	17	19

NATURAL SINES.

	0'	10'	20'	30'	40'	50'	1 2 3	4 5 6	7 8 9
45°	7071	7092	7112	7133	7153	7173	2 4 6	8 10 12	14 16 18
46	7193	7214	7234	7254	7274	7294	2 4 6	8 10 12	14 16 18
47	7314	7333	7353	7373	7392	7412	2 4 6	8 10 12	14 16 18
48	7431	7451	7470	7490	7509	7528	2 4 6	8 10 12	13 15 17
49	7547	7566	7585	7604	7623	7642	2 4 6	8 9 11	13 15 17
50	7660	7679	7698	7716	7735	7753	2 4 6	7 9 11	13 15 17
51	7771	7790	7808	7826	7844	7862	2 4 5	7 9 11	13 14 16
52	7880	7898	7916	7934	7951	7969	2 4 5	7 9 11	12 14 16
53	7986	8004	8021	8039	8056	8073	2 3 5	7 9 10	12 14 16
54	8090	8107	8124	8141	8158	8175	2 3 5	7 8 10	12 14 15
55	8192	8208	8225	8241	8258	8274	2 3 5	7 8 10	12 13 15
56	8290	8307	8323	8339	8355	8371	2 3 5	6 8 10	11 13 14
57	8387	8403	8418	8434	8450	8465	2 3 5	6 8 9	11 12 14
58	8480	8496	8511	8526	8542	8557	2 3 5	6 8 9	11 12 14
59	8572	8587	8601	8616	8631	8646	1 3 4	6 7 9	10 12 13
60	8660	8675	8689	8704	8718	8732	1 3 4	6 7 9	10 11 13
61	8746	8760	8774	8788	8802	8816	1 3 4	6 7 8	10 11 12
62	8829	8843	8857	8870	8884	8897	1 3 4	5 7 8	9 11 12
63	8910	8923	8936	8949	8962	8975	1 3 4	5 6 8	9 10 12
64	8988	9001	9013	9026	9038	9051	1 3 4	5 6 8	9 10 11
65	9063	9075	9088	9100	9112	9124	1 2 4	5 6 7	8 10 11
66	9135	9147	9159	9171	9182	9194	1 2 3	5 6 7	8 9 10
67	9205	9216	9228	9239	9250	9261	1 2 3	4 6 7	8 9 10
68	9272	9283	9293	9304	9315	9325	1 2 3	4 5 6	7 9 10
69	9336	9346	9356	9367	9377	9387	1 2 3	4 5 6	7 8 9
70	9397	9407	9417	9426	9436	9446	1 2 3	4 5 6	7 8 9
71	9455	9465	9474	9483	9492	9502	1 2 3	4 5 6	6 7 8
72	9511	9520	9528	9537	9546	9555	1 2 3	4 4 5	6 7 8
73	9563	9572	9580	9588	9596	9605	1 2 2	3 4 5	6 7 7
74	9613	9621	9628	9636	9644	9652	1 2 2	3 4 5	5 6 7
75	9659	9667	9674	9681	9689	9696	1 1 2	3 4 4	5 6 7
76	9708	9710	9717	9724	9730	9737	1 1 2	3 3 4	5 5 6
77	9744	9750	9757	9763	9769	9775	1 1 2	3 3 4	4 5 6
78	9781	9787	9793	9799	9805	9811	1 1 2	2 3 3	4 5 5
79	9816	9822	9827	9833	9838	9843	1 1 2	2 3 3	4 4 5
80	9848	9853	9858	9863	9868	9872	0 1 1	2 2 3	3 4 4
81	9877	9881	9886	9890	9894	9899	0 1 1	2 2 3	3 3 4
82	9903	9907	9911	9914	9918	9922	0 1 1	2 2 2	3 3 3
83	9925	9929	9932	9936	9939	9942	0 1 1	1 2 2	3 3 3
84	9945	9948	9951	9954	9957	9959	0 1 1	1 1 2	2 2 2
85	9962	9964	9967	9969	9971	9974	0 0 1	1 1 1	2 2 2
86	9976	9978	9980	9981	9983	9985	0 0 1	1 1 1	1 1 2
87	9986	9988	9989	9990	9992	9993	0 0 0	1 1 1	1 1 1
88	9994	9995	9996	9997	9997	9998	0 0 0	0 0 0	1 1 1
89	9998	9999	9999	1.0000	1.0000	1.0000	0 0 0	0 0 0	0 0 0

NATURAL TANGENTS.

(In most cases it is sufficiently accurate to use the values to four figures.)

Angle.	.0 =.0'.	.1 =.6'.	.2 =.12'.	.3 =.18'.	.4 =.24'.	.5 =.30'.	.6 =.36'.	.7 =.42'.	.8 =.48'.	.9 =.54'.
0°	.00000	.00174	.00349	.00524	.00698	.00873	.01047	.01222	.01396	.01571
1	.01745	.01920	.02095	.02269	.02444	.02619	.02793	.02968	.03143	.03317
2	.03492	.03667	.03842	.04016	.04191	.04365	.04541	.04716	.04891	.05066
3	.05241	.05416	.05591	.05766	.05941	.06116	.06291	.06467	.06642	.06817
4	.06993	.07168	.07343	.07519	.07695	.07870	.08046	.08221	.08397	.08573
5	.08749	.08925	.09101	.09277	.09453	.09629	.09805	.09981	.10158	.10334
6	.10510	.10687	.10863	.11040	.11217	.11394	.11570	.11747	.11924	.12101
7	.12278	.12456	.12633	.12810	.12988	.13165	.13343	.13520	.13698	.13876
8	.14054	.14232	.14410	.14588	.14767	.14945	.15124	.15302	.15481	.15660
9	.15838	.16017	.16196	.16376	.16555	.16734	.16914	.17093	.17273	.17453
10	.17633	.17813	.17993	.18173	.18353	.18534	.18714	.18895	.19076	.19257
11	.19438	.19619	.19800	.19982	.20164	.20345	.20527	.20709	.20891	.21073
12	.21256	.21438	.21621	.21803	.21986	.22169	.22353	.22536	.22719	.22903
13	.23087	.23271	.23455	.23639	.23823	.24008	.24192	.24377	.24562	.24747
14	.24933	.25118	.25304	.25490	.25676	.25862	.26048	.26234	.26421	.26608
15	.26795	.26982	.27169	.27357	.27545	.27732	.27920	.28109	.28297	.28486
16	.28675	.28863	.29053	.29242	.29432	.29621	.29811	.30001	.30192	.30382
17	.30573	.30764	.30955	.31146	.31338	.31530	.31722	.31914	.32106	.32299
18	.32492	.32685	.32878	.33072	.33266	.33459	.33654	.33848	.34043	.34238
19	.34433	.34628	.34824	.35019	.35216	.35412	.35608	.35805	.36002	.36199
20	.36397	.36595	.36793	.36991	.37190	.37388	.37587	.37787	.37986	.38186
21	.38386	.38587	.38787	.38988	.39189	.39391	.39593	.39795	.39997	.40200
22	.40403	.40606	.40809	.41013	.41217	.41421	.41626	.41831	.42036	.42242
23	.42447	.42654	.42860	.43067	.43274	.43481	.43689	.43897	.44105	.44314
24	.44523	.44732	.44942	.45152	.45362	.45573	.45784	.45995	.46206	.46418
25	.46631	.46843	.47056	.47270	.47483	.47697	.47912	.48127	.48342	.48557
26	.48773	.48989	.49206	.49423	.49640	.49858	.50076	.50295	.50514	.50733
27	.50953	.51173	.51393	.51614	.51835	.52057	.52279	.52501	.52724	.52947
28	.53171	.53395	.53620	.53844	.54070	.54296	.54522	.54748	.54975	.55203
29	.55431	.55659	.55888	.56117	.56347	.56577	.56808	.57039	.57271	.57502
30	.57735	.57968	.58201	.58435	.58670	.58904	.59140	.59376	.59612	.59849
31	.60086	.60324	.60562	.60801	.61040	.61280	.61520	.61761	.62003	.62245
32	.62487	.62730	.62973	.63217	.63462	.63707	.63953	.64199	.64446	.64693
33	.64941	.65189	.65438	.65688	.65938	.66189	.66440	.66692	.66944	.67197
34	.67451	.67705	.67960	.68215	.68471	.68728	.68985	.69243	.69502	.69761
35	.70021	.70281	.70542	.70804	.71066	.71329	.71593	.71857	.72122	.72388
36	.72654	.72921	.73189	.73457	.73726	.73996	.74267	.74538	.74810	.75082
37	.75355	.75629	.75904	.76180	.76456	.76733	.77010	.77289	.77568	.77848
38	.78129	.78410	.78692	.78975	.79259	.79544	.79829	.80115	.80402	.80690
39	.80978	.81268	.81558	.81849	.82141	.82434	.82727	.83022	.83317	.83613
40	.83910	.84208	.84507	.84806	.85107	.85408	.85710	.86014	.86318	.86623
41	.86929	.87236	.87543	.87852	.88162	.88474	.88784	.89097	.89410	.89725
42	.90040	.90357	.90674	.90993	.91313	.91633	.91955	.92277	.92601	.92926
43	.93252	.93578	.93906	.94235	.94565	.94896	.95229	.95562	.95897	.96232
44	.96569	.96907	.97246	.97586	.97927	.98270	.98613	.98958	.99304	.99651

NATURAL TANGENTS

(In most cases it is sufficiently accurate to use the values to four figures.)

Angle.	.0 = 0°.	.1 = 6'.	.2 = 12'.	.3 = 18'.	.4 = 24'.	.5 = 30'.	.6 = 36'.	.7 = 42'.	.8 = 48'.	.9 = 54'.
45°	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319
46	1.0355	1.0391	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3174	1.3222
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4123	1.4176	1.4229
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5223	1.5282	1.5340
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6318	1.6383	1.6447	1.6512	1.6577
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251
60	1.7320	1.7390	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542
63	1.9626	1.9711	1.9797	1.9883	1.9969	2.0057	2.0145	2.0233	2.0323	2.0412
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2147	2.2251	2.2355
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445
67	2.3558	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4382	2.4504	2.4627
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5781	2.5916
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7033	2.7179	2.7326
70	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8396	2.8555	2.8716	2.8878
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812
76	4.0108	4.0403	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972
77	4.3315	4.3662	4.4015	4.4373	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140
80	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572
84	9.5144	9.6788	9.8448	10.0119	10.1999	10.3885	10.5799	10.7800	10.9888	11.2055
85	11.4301	11.6644	11.9009	12.1603	12.4299	12.7066	12.9966	13.3000	13.6177	13.9511
86	14.3011	14.6699	15.0566	15.4604	15.8955	16.3550	16.8329	17.3433	17.8866	18.4644
87	19.0811	19.7400	20.4466	21.2055	22.0222	22.9004	23.8559	24.8968	26.0311	27.2711
88	28.6366	30.1455	31.8211	33.6699	35.8011	38.1888	40.9177	44.0666	47.7400	52.0811
89	57.2900	63.6577	71.6155	81.8477	95.4899	114.5999	143.3244	190.9888	285.4888	572.7666

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